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Solving Fuzzy Transportation Problem by Using Ranking Function with Triangular Membership

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Introduction

The fuzzy set theory that discovered by Zadeh in ١٩٦٥ is the technique through which determine the degree of membership or the degree of health, which is the extent of scores between right and wrong, and this is the difference between him and Boolean logic (binary), who only knows right and wrong (True - False).

The fuzzy transportation problem is referring to transportation problem with the all or some coefficients are fuzzy number and the aim is to find a cost less transfer units available in the sources when they are uncertain (fuzzy).

Many scientists have studied the raking functions of the order numbers to resolve fuzzy in various fields, functions ranks fuzzy change numbers to the real numbers. The functions can be applied to fuzzy transportation problems when the costs (c_{kl}), demand (D_{kl}) or supply (s_{kl}) are fuzzy numbers.

This thesis is organized as follows:-

Chapter one: - presented the literature review as well as fuzzy set theory, some priorities of fuzzy relationship and explain the transportation problems.

Chapter two: - studies fuzzy transportation problem, Ranking function and we proposed two ranking functions and apply to fuzzy transportation problem; we also presented two examples and conduct compare between the results of ranking function.

Chapter three: - studies the application of the fuzzy transportation problem to the real data from "Oil Products Distribution Company" and we apply the algorithms that have been studied in this letter and then compared to the results.

Finally, in **Chapter four: -** discussed the results and recommendations for the company is presented.

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١-١ Introduction

The fuzzy set theory solved many phenomena of life that does not rely on the principle of true or false. This chapter consists of (١-٢) the aim of thesis. In section (١-٣) we introduce a historical review of some scientists who have studied and solved fuzzy transportation problem by using ranking function. In section (١-٤) fuzzy set theory, some concepts and important operations are explained by examples. In section (١-٥) fuzzy relation and some examples are studied. In section (١-٦) transportation problem is explained.

١-٢ The Aim of Thesis

The target of this thesis is inspecting and studying fully fuzzy transportation problem when the demand, supply and transportation cost are fuzzy, using many algorithms for ranking function, as well as using a new two proposed algorithms for ranking functions.

Then, we compare between these algorithms for ranking function and new two proposed algorithms through using total cost for fully fuzzy transportation problem.

١-٣ Literature Review

Many authors studied ranking function to solve the problem of fuzzy transportation.

- Chanas, Kolodziejczyk and Machaj (١٩٨٤) presented a fuzzy linear programming model for solving fuzzy transportation problem. **[٧]**

- Chanas and Kuchta (١٩٩٦) proposed method found optimal solution for the fuzzy transportation problem when only the coefficient cost was fuzzy numbers. **[٦]**

- Liu and Kao (٢٠٠٤) described a method to solve a fuzzy transportation problem based on extension principle. **[١٨]**

- Chiang J. (٢٠٠٥) proposed an algorithm for ranking function when the demand and supply are fuzzy numbers only; he found two different triangular memberships for demand and supply. **[٨]**

- Nagoor Gani and Abdul Razak (٢٠٠٦) obtained a fuzzy solution for a two stage cost minimizing fuzzy transportation problem in which supplies and demands are trapezoidal fuzzy number. **[٢١]**

- Basirzadeh and Abbasi (٢٠٠٨) proposed a new method to solve the fuzzy transportation problem using ranking function based on $\alpha -$ cut. **[٣]**

- Lin and Tsai (٢٠٠٩) used a two stage genetic algorithm for solving the transportation problem when the demands and supplies were fuzzy numbers. **[١٧]**

- Pandian and Natarajan (٢٠١٠) proposed a fuzzy zero point method for finding a fuzzy optimal solution for fuzzy transportation problem where all parameters are trapezoidal fuzzy numbers. **[٢٦]**

- Basirzadeh (٢٠١١) solved the fuzzy transportation problem depending on the ranking function of Yager (١٩٨١) which found ranking function of trapezoidal and triangular memberships. **[٤]**

- Sudhakar and Kumar (٢٠١١) studied different method to solve two stage fuzzy transportation problems. **[٣٣]**

- Poonam S., Abbas and Gupta (٢٠١٢) presented a ranking technique with α-cut for solving fuzzy transportation problem, where the demand and supply are triangular fuzzy numbers. **[٢٧]**

- Mohanaselvi and Ganesan (٢٠١٢) proposed a new method for the initial fuzzy feasible solution to a fully fuzzy transportation problem. Then used modified distribution method to find fuzzy optimal solution for fully transportation problem without convert to a crisp problem. **[٢٠]**

- Ramesh Kumar and Murugesan (٢٠١٢) presented a modified revised simplex method for minimizing fuzzy transportation problem of triangular fuzzy numbers. **[٢٩]**

- Nagoor Gani and Abbas (٢٠١٣) used the idea of chiang J. at (٢٠٠٥) and studied that the demand and supply are fuzzy numbers only depending are two different triangular memberships. **[٢٢]**

- Narayanamoorthy and Saranya and S.Maheswari, (٢٠١٣) proposed algorithm called Russell's method to solved fuzzy transportation problem which are used for any kind fuzzy numbers. **[٢٣]**

- Naresh kumar and Kumara (٢٠١٤) proposed method, where the cost, demand and supply are symmetric triangular fuzzy numbers, then they developed fuzzy version of Vogel's algorithm for finding fuzzy optimal solution of fuzzy transportation problem.**[٢٤]**

- Khalaf (٢٠١٤) proposed a new approach titled fuzzy Russell's approximation has been developed for solving fuzzy transportation problem when the only cost coefficients is fuzzy number. **[١٤]**

١-٤ Fuzzy Set Theory

The scientist Aristotle put the binary logic three hundred years B.C which is regarded as a basic concept in mathematics, that linked with ordinary set theory depending on {٠,١}, which means mathematically, if the element belong to a set, then the membership is one, but if not belong to a set then the membership is zero. For this reason, the ordinary set theory is depending on binary logic.

Zadeh in ١٩٦٥ noticed that the binary logic cannot represented all phenomena in life because many phenomena in life is uncertain which means mathematically that the element belongs to the set as a certain percent and compute from membership function depending on [٠,١].

The fuzzy set theory utilizing at all phenomenon's in life includes the request for an employee to work for the company. The conditions laid down were: that the job seeker is (fluent in English), (be very good at using the computer), (have a decent appearance) and (has a good process in the field of marketing experience).

Note that all items such as "fluent English" and "fluent in the use of computers" and "a decent appearance" and "a good experience" ... etc. all are not clear value definitions of true or false, but ranging health blurry values and the factors that have been collecting these items is a logical factor. such as blurry "very" and "and" . evaluating any student for the job placed his mark (between zero and one) for each of the items, and using logical operators, we get the alignment mark of the office and the differentiation between the final marks that we get from all applicants to be the owner of the highest value is the desired employee.

On the other hand, if we insist on developing conditions a way that makes it possible to use ordinary logic, we will have first to establish clear value definitions of true or false, such as "holds a TOEFL certificate in the English language" and "holds a certain degree in computer science" ... etc., which causes injustice for a lot of people as well as they do not give us a clear way to differentiate between the different applicants and ultimately it's up to the mood, director of human resources at the company.

The last example is from the field of weather, it is clear that the phrase "nice weather" has a fuzzy value have graded the weather is very bad to very nice, plus their vehicle include many factors phrase, the weather spectrum means, for example, that the atmosphere is acceptable and that the temperature is moderate and the humidity is low and that the wind will be moderate. It is clear that all these values are fuzzy and therefore the previous statement form fuzzy equation and many other examples, including in the field of medicine, sports and competitions, and in many areas of life.

Definition [٩]

Let Ω be a nonempty set (universal set). A fuzzy set $\widetilde{A}$ in Ω is characterized by its membership function

$$\mu_{\widetilde{A}} : \Omega \to [٠,١]$$

and $\mu_{\widetilde{A}(a)}$ is the interpreted as a degree of membership of element a in fuzzy set A for each $a \in \Omega$ and denoted for its set by $\widetilde{A}$.

$$\widetilde{A} = \{(a, \mu_{\widetilde{A}(a)}) : a \in \Omega\}.$$

Definition [٢٣]

A crisp set is a special case of a Fuzzy set, in which the membership function has only two values, ٠ and ١.

Remark [٩]

If $\Omega = \{a_١, \ldots, a_n\}$ is a finite set and $\tilde{A}$ is a fuzzy set in Ω, then we often use the notation

$$A=\frac{\mu_{\tilde{A}}(a_i)}{a_i}+\ldots+\frac{\mu_{\tilde{A}}(a_n)}{a_n}.$$

where the term $\frac{\mu_{\tilde{A}}(a_i)}{a_i}$, i=١,…,n signifies that $\mu_{\tilde{A}}(a_i)$ is the graded of membership of a_i in A, and the plus sign represented the union.

١-٤-١ Some Concepts of Fuzzy Set

We will present some concepts of the fuzzy set, we start by defining Ω which is the universal set and let $\tilde{A}$ be a fuzzy subset of Ω with membership $\mu_{\tilde{A}}(a)$.

*** The support of $\tilde{A}$ [٣٥]**

The support of a fuzzy set $\tilde{A}$ (supp ($\tilde{A}$)) is the crisp set of all $a \in \Omega$,

$\mu_{\tilde{A}}(a) > ٠$ i.e. supp ($\tilde{A}$) = $\{a \in \Omega : \mu_{\tilde{A}}(a) > ٠\}$

*** α −Level set (α −cut set) [٣٥]**

The crisp set of elements that belong to the fuzzy set $\tilde{A}$ at not less than the degree α is called the α −level set, and $\alpha \in [٠,١]$.

$A_\alpha = \{ a \in \Omega : \mu_{\widetilde{A}(a)} \geq \alpha\}$,

$\grave{A}_\alpha = \{ a \in \Omega : \mu_{\widetilde{A}(a)} > \alpha\}$ is called strong α −cut set.

- Example (١.١)

Let $\Omega = \{١,٢,٣,٤\}$ be a universal set, consider membership of fuzzy set

$$\mu_{\widetilde{A}(a)} = \frac{١}{a^٢+١}$$

$\widetilde{A} = \{(١,٠.٥),(٢,٠.٢),(٣,٠.١),(٤,٠.٠٦)\}$.

The supp $(\widetilde{A}) = \{١, ٢, ٣, ٤\}$

Let $\alpha =$ ٠.٢

$$A_\alpha = \{١,٢\}, \quad \grave{A}_\alpha = \{١\}$$

*** Normality [٥]**

The fuzzy set $\widetilde{A}$ is normal if its core is nonempty

Equivalently; we can find at least one element $a \in \Omega$ s.t $\mu_{\widetilde{A}(a)} = ١$.

*** Kernel of fuzzy set [١٩]**

The kernel of fuzzy set $\widetilde{A}$ is the crisp set

$$\text{Ker}\ (\widetilde{A}) = \{a \in \Omega : \mu_{\widetilde{A}(a)} = ١\}.$$

* We can called the fuzzy set $\widetilde{A}$ is normal if it's Ker $(\widetilde{A}) \neq \emptyset$.

*** Example (١.٢)**

Let X= {-٢,-١, ٠, ١, ٢} define the fuzzy set $\widetilde{A}$ with membership function:-

$$\mu_{\tilde{A}}(a) = \frac{١}{(٢a+١)^٢}$$

$\tilde{A}$={(-٢,٠.١١),(-١,١),(٠,١),(١,٠.١١),(٢,٠.٠٤)}.

$\tilde{A}$ is normal

Ker ($\tilde{A}$) = {-١, ٠}.

*** Crossover point [٥]**

A crossover point of a fuzzy set $\tilde{A}$ is a point a ∈ Ω s.t $\mu_{\tilde{A}}(a)$=٠.٥

*** Fuzzy singleton [٥]**

A fuzzy set whose support is a single point in Ω with $\mu_{\tilde{A}}(a)$ = ١ is called fuzzy singleton.

*** Example (١.٣)**

Let Ω ={١, ٢, ٣}; consider the membership function

$$\mu_{\tilde{A}}(a) = \begin{cases} \frac{١}{٢a^٢} & a \in \Omega \\ ٠ & o.w \end{cases}$$

Then $\tilde{A}$ ={(١, ٠.٥), (٢, ٠.١٢٥), (٣, ٠.٠٥٥)}

The crossover point of $\tilde{A}$ is {١}; i.e. $\mu_{\tilde{A}}(١)$ = ٠.٥

$\tilde{A}$ is not fuzzy singleton

*** The hight of fuzzy set [١٥]**

The hight of fuzzy set $\tilde{A}$ is the largest membership grade attained by any element in fuzzy set.

* The empty fuzzy subset of Ω is defined as the fuzzy subset ∅ of Ω s.t ∅(a) =٠ ∀ a ∈ Ω. **[١٢]**

*** Example (١.٤)**

Let $\Omega = \Re$, we define a membership function for fuzzy set of real number

$\widetilde{A}$ ={Real number near ٢} as follows:-

$$\mu_{\widetilde{A}(a)} = \frac{١}{١+(a-٢)^{٢}} .$$

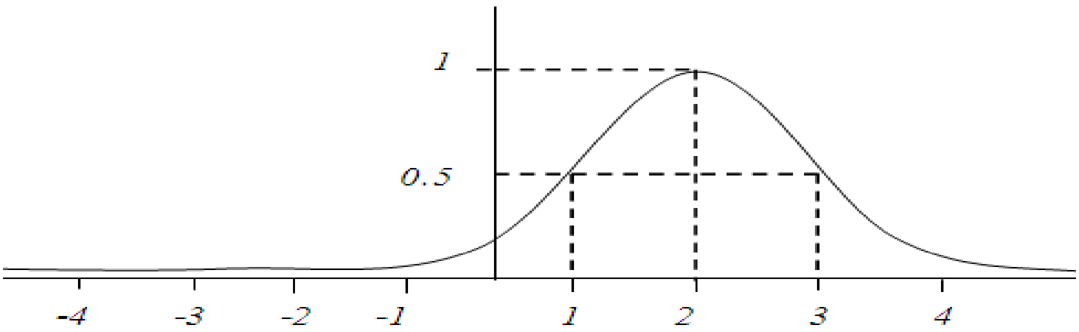

Figure (١-١) represents membership function near ٢.

$\widetilde{A}$ is normal, since its kernel={٢}

Supp ($\widetilde{A}$) $= \Re$.

Crossover point of $\widetilde{A}$= {١, ٣}.

The hight of $\widetilde{A}$ is (١) is not fuzzy singleton.

*** Convex fuzzy set [١٥]**

The fuzzy set $\widetilde{A}$ is convex iff its α −cut is convex.

In other words, we may say that a fuzzy set $\widetilde{A}$ is convex iff

$$\mu_{\widetilde{A}}(\lambda a + (١ - \lambda)b) \geq \min[\mu_{\widetilde{A}(a)}, \mu_{\widetilde{A}(b)}] . \forall\, a, b \in \boldsymbol{\Omega} \text{ and } \forall\, \lambda \in [٠,١].$$

*** Fuzzy number [٩]**

The fuzzy number $\widetilde{A}$ is a fuzzy set of the real line with a normal, fuzzy convex and continuous membership function of bounded support.

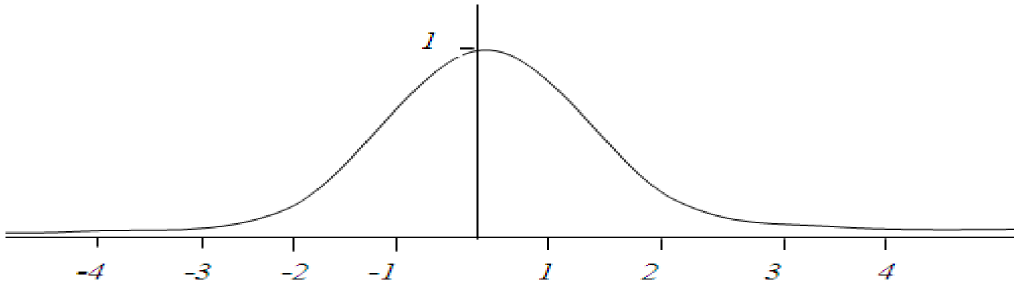

Figure (١-٢) represents fuzzy number

١-٤-٢ Some Operation of Fuzzy Set

We can define some operations of fuzzy sets by using two fuzzy sets $\tilde{A}$ and $\tilde{B}$ with membership $\mu_{\tilde{A}}(a)$ and $\mu_{\tilde{B}}(b)$, and let Ω be a universal set.

***The complement of fuzzy set** $\tilde{A}$ denoted by $\tilde{A}^c$ is defined as

$$\mu_{\tilde{A}^c}(a) = ١ - \mu_{\tilde{A}}(a). \quad a \in \Omega. \ \textbf{[١٢]}$$

* **The intersection of two fuzzy sets** $\tilde{A}$ and $\tilde{B}$ is a fuzzy set $\tilde{D}$, written as $\tilde{D} = \tilde{A} \cap \tilde{B}$ or $\tilde{D} = \tilde{A}$ and $\tilde{B}$ whose membership functionis:

$$\mu_{\tilde{D}}(d) = \text{Min}\,\{\mu_{\tilde{A}}(a), \mu_{\tilde{B}}(b)\}. \ \textbf{[٥]}$$

* **The union of two fuzzy sets** $\tilde{A}$ and $\tilde{B}$ is a fuzzy set $\tilde{U}$, written as

$\tilde{U} = \tilde{A} \cup \tilde{B}$ Or $\tilde{U} = \tilde{A}$ or $\tilde{B}$ whose membership function

$$\mu_{\tilde{U}}(u) = \text{Max}\left\{\mu_{\tilde{A}}(a), \mu_{\tilde{B}}(b)\right\}. \ \textbf{[٥]}$$

* **Example (١.٥)**

Let Ω = {١, ٢, ٣, ٤} define $\tilde{A}$ and $\tilde{B}$ as follows

$\tilde{A}$={(١,٠.٢),(٢,٠.٦),(٣,٠.٥),(٤,٠.٧)}.

$\tilde{B}$={(١,٠.٥),(٢,٠.٤),(٣,٠.٣),(٤,٠.٩)}.

$\widetilde{A}^c$={(١,٠.٨),(٢,٠.٤),(٣,٠.٥),(٤,٠.٣)}.

$\widetilde{B}^c$ ={(١,٠.٥),(٢,٠.٦),(٣,٠.٧),(٤,٠.١)}.

$\widetilde{D}$ ={(١,٠.٢),(٢,٠.٤),(٣,٠.٣),(٤,٠.٧)}.

$\widetilde{U}$ ={(١,٠.٥),(٢,٠.٦),(٣,٠.٥),(٤,٠.٩)}.

*** Algebraic sum [١٦]**

$\widetilde{A} + \widetilde{B}$ is defined as $\mu_{\widetilde{A}+\widetilde{B}}(a+b) = \mu_{\widetilde{A}}(a) + \mu_{\widetilde{B}}(b) - \mu_{\widetilde{A}}(a) \cdot \mu_{\widetilde{B}}(b)$.

*** Bounded sum [١٦]**

$\widetilde{A} \oplus \widetilde{B}$ is defined as $\forall a \in \boldsymbol{\Omega}$ $\mu_{\widetilde{A}\oplus\widetilde{B}}(a\oplus b)$= min {١, $\mu_{\widetilde{A}}(a) + \mu_{\widetilde{B}}(b)$}.

*** Example (١.٦)**

Let Ω = {٠, ١, ٢} define $\widetilde{A}$ and $\widetilde{B}$

$\widetilde{A}$ ={(٠, ٠.٣), (١, ٠.٥), (٢, ٠.٦)}.

$\widetilde{B}$ ={(٠, ٠.٦), (١, ٠.٧), (٢, ٠.٤)}.

$\widetilde{A} + \widetilde{B}$ ={(٠, ٠.٧٢), (١, ٠.٨٥), (٢, ٠.٧٦)}.

$\widetilde{A} \oplus \widetilde{B}$= {(٠, ٠.٩), (١, ١), (٢, ١)}.

*** Difference in fuzzy sets [١٦]**

The difference of $\widetilde{A}$ and $\widetilde{B}$ is $\widetilde{A} - \widetilde{B} = \widetilde{A} \cap \widetilde{B}^c$.

*** Bounded difference [١٦]**

Dented for bounded difference by $\ominus$ and define membership function by

$\mu_{\widetilde{A}\ominus\widetilde{B}}$ =Max {٠,$\mu_{\widetilde{A}}(a) - \mu_{\widetilde{B}}(b)$}.

*** Example (١.٧)**

Let $\Omega = \{١, ٢, ٣\}$

$\widetilde{A} = \{(١, ٠.٦), (٢, ٠.٥), (٣, ٠.٧)\}$.

$\widetilde{B} = \{(١, ٠.٤), (٢, ٠.٦), (٣, ٠.٨)\}$.

$\widetilde{A} - \widetilde{B} = \{(١, ٠.٦), (٢, ٠.٤), (٣, ٠.٢)\}$.

$\widetilde{A} \ominus \widetilde{B} = \{(١, ٠.٢), (٢, ٠), (٣, ٠)\}$.

*** Algebraic product [١٦]**

Denoted by ($\cdot$) and define membership function by

$\mu_{\widetilde{A}\cdot\widetilde{B}}(a\cdot b) = \mu_{\widetilde{A}}(a) \cdot \mu_{\widetilde{B}}(b)$, $\forall$ a, b $\in \Omega$.

*** Bounded product [١٦]**

It's dented by $\odot$ and defines membership function by

$\mu_{\widetilde{A}\odot\widetilde{B}}(a\odot b) = \mathrm{Max}\,\{٠, \mu_{\widetilde{A}}(a) + \mu_{\widetilde{B}}(b) - ١\}$, $\forall$ a, b$\in \Omega$.

*** Power of fuzzy set [١٦]**

Second power of fuzzy set $\widetilde{A}$ is defined as follows:-

$\mu_{\widetilde{A}^{٢}}(a) = [\mu_{\widetilde{A}}(a)]^{٢}$.

Similarly k^{th}power of fuzzy set $\widetilde{A}^{k}$ may be computed as,

$\mu_{\widetilde{A}^{k}}(a) = [\mu_{\widetilde{A}}(a)]^{k}$.

*** Example (١.٨)**

We define a possible membership function for fuzzy set $\widetilde{A}$.

Let $\Omega = \{١, ٢, ٣, ٤, ٥\}$

$\mu_{\tilde{A}(a)} = \frac{a}{a^{٢}+٢}$; $\mu_{\tilde{B}(b)} = \frac{١}{٥b^{٢}}$

$\tilde{A}$ ={(١,٠.٣٣),(٢,٠.٣٣),(٣,٠.٢٧),(٤,٠.٢٢)}.

$\tilde{B}$ ={(١,٠.٢),(٢,٠.٠٥),(٣,٠.٠٢),(٤,٠.٠١)}.

$\tilde{A} \cdot \tilde{B}$={(١,٠.٠٦٦),(٢,٠.٠١٦),(٣,٠.٠٠٥),(٤,٠.٠٠٢)}.

$\tilde{A} \odot \tilde{B}$={(١,٠),(٢,٠),(٣,٠),(٤,٠)}.

$\tilde{A}^{٢}$={(١,٠.١٠٩),(٢,٠.١٠٩),(٣,٠.٠٧٣),(٤,٠.٠٤٨)}.

$\tilde{B}^{٢}$={(١,٠.٠٤),(٢,٠.٠٠٢٥),(٣,٠.٠٠٠٤)(٤,٠.٠٠٠١)}.

*** Containment or subset [٥]**

Fuzzy set $\tilde{A}$ is contained in fuzzy set $\tilde{B}$ (or equivalently; $\tilde{A}$ is subset of $\tilde{B}$) , $\tilde{A} \subseteq \tilde{B}$) iff $\mu_{\tilde{A}(a)} \leq \mu_{\tilde{B}(b)}$ $\forall$ a, b$\in \Omega$.

*** Equality of fuzzy sets [٩]**

Let $\tilde{A}$ and $\tilde{B}$ are fuzzy subsets of classical set Ω . $\tilde{A}$ and $\tilde{B}$ are said to be equal denoted by $\tilde{A} = \tilde{B}$ if $\tilde{A} \subset \tilde{B}$ and $\tilde{B} \subset \tilde{A}$.

In other words, $\tilde{A} = \tilde{B}$ iff $\mu_{\tilde{A}(a)} = \mu_{\tilde{B}(b)}$ $\forall$ a, b $\in \Omega$.

*** Example (١.٩)**

Let Ω ={٠,١,٢,٣,٤} and A$\subset \Omega$, A={٠,١,٢} , A^c={٣,٤}

It's clear that in crisp case A$\cup$ A^c= Ω and A$\cap$ $A^c = \emptyset$

In case fuzzy set define $\tilde{A}$ by

$\tilde{A}$ ={(٠,٠.٢),(١,٠.٥),(٢,٠.٣),(٣,٠.٧),(٤,٠.٨)}.

$\tilde{A}^c$ ={(٠,٠.٨),(١,٠.٥),(٢,٠.٧),(٣,٠.٣),(٤,٠.٢)}.

$\widetilde{A} \cup \widetilde{A}^c$={(٠,٠.٨),(١,٠.٥),(٢,٠.٧),(٣,٠.٧),(٤,٠.٨)} $\neq \Omega$

$\widetilde{A} \cap \widetilde{A}^c$ ={(٠.٠.٢), (١, ٠.٥), (٢, ٠.٣), (٣, ٠.٣), (٤, ٠.٢)} $\neq \emptyset$

١-٤-٣ The Types Membership Function

The membership function in fuzzy set theory plays an important and a basic rule to determine the belong degree of elements to the certain fuzzy set.

There are two kinds of membership functions:

١) Linear membership function

a) LR fuzzy number [٢٥]

A fuzzy number $\widetilde{A}$ on $\Re$ is said to be LR fuzzy number if there exists a real number x and a, b, c ≥ ٠ s.t

$$\mu_{\widetilde{A}(x)} = \begin{cases} L\left(\frac{a-x}{b}\right) & x \leq b \\ R\left(\frac{x-a}{c}\right) & x \geq b \end{cases}$$

In which L(x) and R(x) are continuous and non-decreasing functions on the real number line.

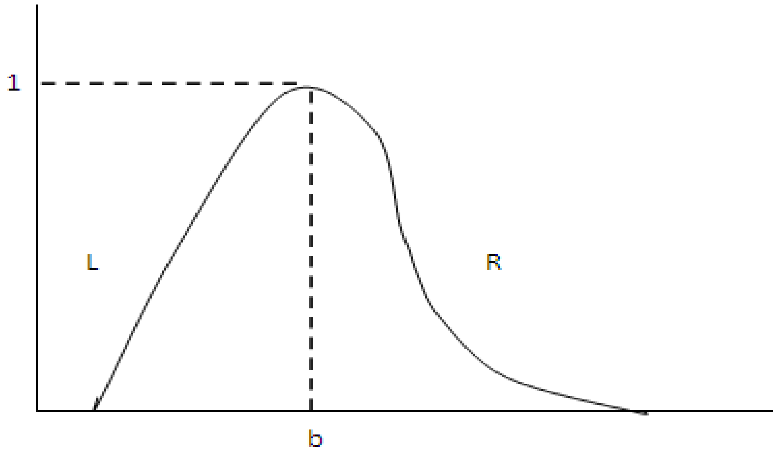

Figure (١-٣) represents LR fuzzy number

b) Triangular Fuzzy Number [٤]

A fuzzy number $\widetilde{A}$ is a triangular fuzzy number denoted by (a, b, c) where a, b and c are real numbers and their membership function $\mu_{\widetilde{A}(a)}$ is given by:-

$$\mu_A(x) = \begin{cases} \frac{x-a}{b-a} & a \le x \le b \\ ١ & x = b \\ \frac{c-x}{c-b} & b \le x \le c \end{cases}$$

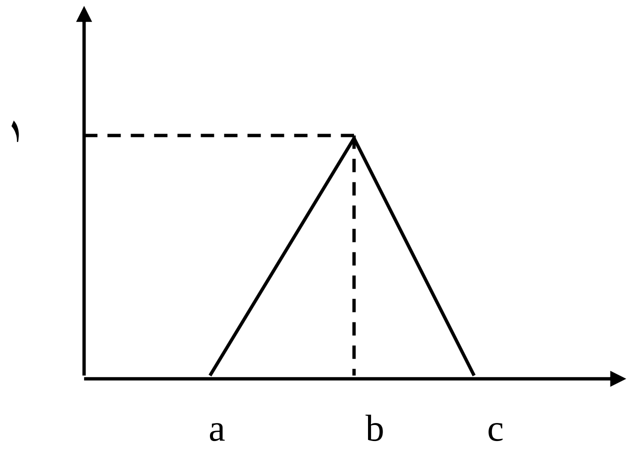

Figure (١-٤) represents triangular fuzzy number

c) Trapezoidal fuzzy number [٣٠]

A fuzzy number $\widetilde{A}$ defined on the universal set of real number denoted by $\widetilde{A}$= (a, b, c, d) is said to be trapezoidal fuzzy number if its membership function $\mu_{\widetilde{A}(x)}$ is given by:-

$$\mu_{\widetilde{A}(x)} = \begin{cases} \dfrac{(x-a)}{(b-a)} & a \le x < b \\ ١ & b \le x \le c \\ \dfrac{(d-x)}{(d-c)} & c \le x \le d \\ ٠ & o.w \end{cases}$$

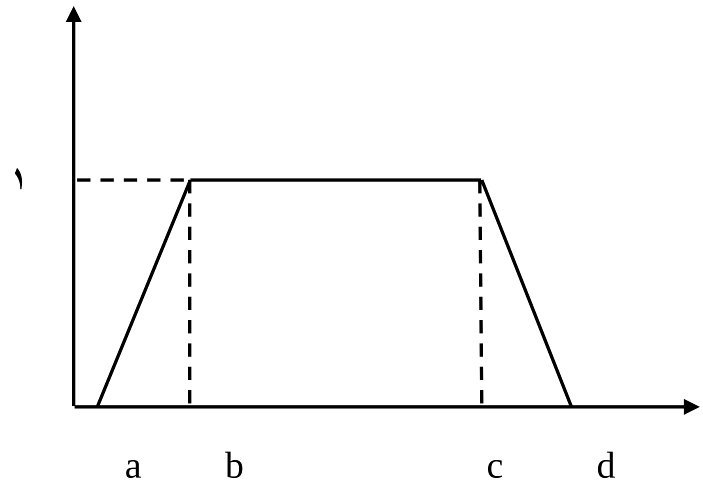

Figure (١-٥) represents the trapezoidal fuzzy number

٢) Nonlinear Membership Function

We offer some of the functions of nonlinear membership function.

a) π_Function [٢]

This function has Roughly Bell shape. The function of this kind is a good alternative triangular function, which has two points coup $c \mp \frac{b}{٢}$ at each side of the function, which has membership function as:-

$$\mu_{\tilde{A}(x)} = \begin{cases} s\left(a, c-b, c-\frac{b}{٢}, c\right) & x \leq c \\ ١ - s\left(a, c+b, c+\frac{b}{٢}, c\right) & x \geq c \end{cases}$$

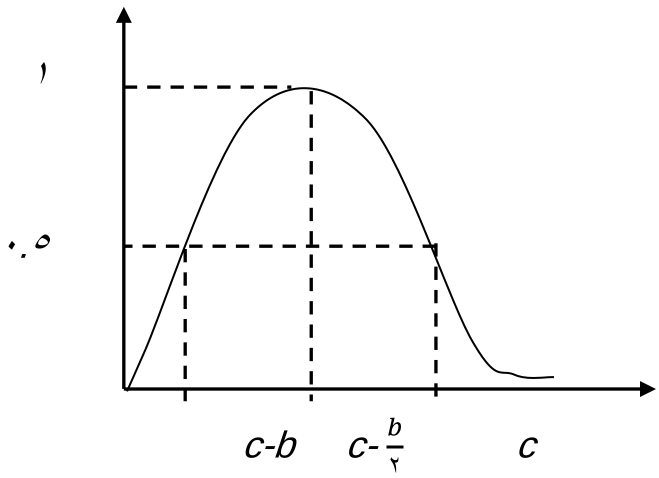

Figure (١-٦) represents π_function

b) Exponential function [٢]

This function of a one parameter k; k > ٠ is defined by:-

$$\mu_{e(a)} = e^{-ka}$$

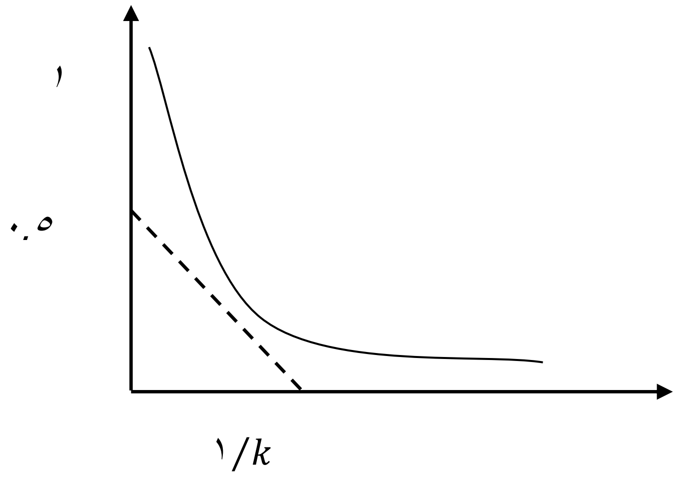

Figure (١-٧) represents Exponential function

١-٥ Fuzzy Relation

If a crisp relation R represents from sets A to B, for x∈ A and y∈ B, its membership function $\mu_{\mathcal{R}(x,y)}$ is,

$$\mu_{R(x,y)} = \begin{cases} ١ & \text{iff } (x,y) \in R \\ ٠ & \text{iff } (x,y) \notin R \end{cases}$$

This membership function maps A× B to set {٠, ١} that is $\mu_{\mathcal{R}}$: A× B →{٠, ١}. **[١١]**

*** Definition [٥]**

Fuzzy relation is a fuzzy subset of X × Y , and is characterized by the membership function. $\mu_{\mathcal{R}(x,y)}$, i.e.

$$R(x,y) = \{((x, y), \mu_{\mathcal{R}(x,y)}) : (x, y) \in X \times Y\}.$$

*** Example (١.١٠)**

Let X= {١, ٢, ٣, ٤} and Y= {٢, ٣} with membership function

$$\mu_{R(x,y)} = \frac{x+y}{(x+y)^{٢}}$$

$$R(x,y) = \{((١,٢),٠.٣٣),((١,٣),٠.٢٥),((٢,٢),٠.٢٥),((٢,٣),٠.٢), ((٣,٢),٠.٢), ((٣,٣),٠.١),((٤,٢),٠.١٦),((٤,٣),٠.١٤)\}.$$

*** Definition (Domain and range of fuzzy relation) [١٦]**

When fuzzy relation R is defined in crisp sets A and B, the domain and range of this relation are defined as:-

$$\mu_{dom_{(R)}(x)} = \max_{y\in B} \mu_{R(x,y)}.$$

$$\mu_{ran_{(R)}(y)} = \max_{x\in A} \mu_{R(x,y)}.$$

Then set A becomes the support of dom(R) and dom (R) $\subseteq$ A, and set B is the support of ran(R) and ran (R) $\subseteq$ B.

١-٥-١ Operations of Fuzzy Relation

We can apply operation of fuzzy sets to the fuzzy relation because relation is kind of set.

*** Union Relation [١٦]**

The union of two relations R and Q is defined as follows:-

$$\forall\, (x, y) \in A \times B\ ,\ \mu_{R \cup Q^{(x,y)}} = \text{Max}\ \{\mu_{R^{(x,y)}}, \mu_{Q^{(x,y)}}\}.$$

*** Intersection Relation [١٦]**

The intersection relation $R \cap Q$ of set A and B is defined by the following membership function:-

$$\mu_{R \cap Q^{(x,y)}} = \text{Min}\ \{\mu_{R^{(x,y)}}, \mu_{Q^{(x,y)}}\},\ \forall\, (x, y) \in A \times B.$$

*** Complement Relation [١٦]**

Complement relation $\overline{R}$ for fuzzy relation R defined as the following membership function:

$$\forall\, (x,y) \in A \times B, \mu_{\overline{R}^{(x,y)}} = ١ - \mu_{R^{(x,y)}}.$$

*** The Maximum-Minimum Composition [١٣]**

Let X, Y and Z be universal sets and let R be a relation that related elements from X to Y

$$R = \{((x, y), \mu_{R(x,y)}): x \in X, y \in Y\};\ R \subseteq X \times Y.$$

and $$Q = \{((y, z), \mu_{Q(y,z)}): y \in Y, z \in Z\};\ Q \subseteq Y \times Z.$$

Then $\mathcal{H}$ will be the relation that relats elements in X that R contains to the elements in Z that Q contain i.e. $\mathcal{H} = R \circ Q$.

The Max-Min composition is defined as the following membership function:-

$$\mu_{\mathcal{H}(x,z)} = \max_{y \in Y}(\min(\mu_{R(x,y)}, \mu_{Q(y,z)}))$$

*** Example (١.١٢)**

Let X={١,٢} , Y={٣,٤} , Z={٤,٥} define two relations with membership function:-

$$\mu_{R(x,y)} = \frac{x}{x+y} \text{ and } \mu_{Q(y,z)} = \frac{z}{z+y^{٢}}$$

R(x, y)={((١,٣),٠.٢٥),((١,٤),٠.٢),((٢,٣),٠.٤),((٢,٤),٠.٣)}.

Q(y, z)={((٣,٤),٠.٣٠),((٣,٥),٠.٣٥),((٤,٤),٠.٢),((٤,٥),٠.٢٤)}.

R∪Q={((٣,٤),٠.٣٠),((٣,٥),٠.٣٥),((٢,٣),٠.٤),((٢,٤),٠.٣)}.

R∩Q={((١,٣),٠.٢٥),((١,٤),٠.٢),((٤,٤),٠.٢),((٤,٥),٠.٢٤)}.

$\overline{R}_{(x,y)}$ ={((١,٣),٠.٧٥),((١,٤),٠.٨),((٢,٣),٠.٦),((٢,٤),٠.٧)}.

$\overline{Q}_{(y,z)}$ ={((٣,٤),٠.٧٠),((٣,٥),٠.٦٥),((٤,٤),٠.٨),((٤,٥),٠.٧٦)}.

$\mathcal{H}$ =R∘ Q=Max{((١,٣),٠.٢٥),((١,٤),٠.٢),((٤,٤),٠.٢),((٤,٥),٠.٢٤)}.

= {((١, ٣), ٠.٢٥)}.

١-٦ The Transportation Problem

The transportation problem is a special class of linear programs that deals with shipping a commodity from sources to destination.

The objective is to determine the shipping schedule that minimizes the total shipping cost while satisfying supply and demand limits. The application of the transportation problem can be extended to other areas of operations, including inventory control, employment scheduling. **[٣٤]**

The origin of transportation problem dates back to ١٩٤١ when Hichcook presented a study entitled "The Distribution of a product from several sources to numerous localities". The presentation is regarded as the first important contribution to the solution of transportation problems.

In ١٩٤٧ Koopmans presented a study called "Optimum utilization of the transportation system ". These two contributions are mainly responsible for the development of transportation problems which involve a number of shipping sources and numbers of destinations. **[١٠]**

١-٦-١ Definition of the Transportation Problem [١٠]

Transportation model deals with problems concerning as to what happens to the effectiveness function when we associate each of a number of sources with each of a possibly different number of destination .The total supply of each source and the total demand of each destination is given and its desire to find how the associations is made as a subject to the limitations on totals. The distinct feature of transportation problem is that sources and destinations must be expressed in terms of only kind of unit. Suppose that there are (m)sources and (n) destinations, let (S_k) be the number of supply units obtainable at sources k (k=١,٢,٣,…,m) and (D_l) be the number of demand units required at destination l (l=١,٢,٣,…,n),and let (c_{kl})represents the unit transportation cost for transporting the units from sources k to destination l .

If x_{kl} ($x_{kl} \geq ٠$) is the number of units transported from sources k to destination l, then the transportation problem is formulated as follows:-

$$\text{Minimize } z= \sum_{k=١}^{m} \sum_{l=١}^{n} c_{kl}\, x_{kl}$$

S.t

$$\sum_{l}^{n} x_{kl} = S_k \qquad k = ١, \dots, m$$

$$\sum_{k}^{m} x_{kl} = D_l \qquad l = ١, \dots, n$$

$$x_{kl} \geq ٠ \qquad k=١ \dots m, \quad l=١ \dots n$$

Table (١-١) represents the supply, demand and cost transportation problem

	١	٢	٣	… l …	n	Supply
١	$c_{١١}$	$c_{١٢}$	$c_{١٣}$	$c_{١l}$	$c_{١n}$	$S_{١}$
٢	$c_{٢١}$	$c_{٢٢}$	$c_{٢٣}$	$c_{٢l}$	$c_{٢n}$	$S_{٢}$
٣	$c_{٣١}$	$c_{٣٢}$	$c_{٣٣}$	$c_{٣l}$	$c_{٣n}$	$S_{٣}$
⋮ k ⋮	$c_{k١}$	$c_{k٢}$	$c_{k٣}$	c_{kl}	c_{kn}	S_k
m	$c_{m١}$	$c_{m٢}$	$c_{m٣}$	c_{ml}	c_{mn}	S_m

Demand	$D_{١}$	$D_{٢}$	$D_{٣}$	D_l	D_n	

The objective of transportation problem is to determine the number of units to be transported from source k to destination l so that the total transportation cost is minimum.

١-٦-٢ Solving Transportation Problem

To solve the transportation problem we must make sure that the total of available of sources is equal to the total of required destination, which is called balanced transportation problem, that is:

$$\sum_{k=١}^{m} S_k = \sum_{l=١}^{n} D_l \ .$$

Otherwise that is if $\sum_{k=١}^{m} S_k \neq \sum_{l=١}^{n} D_l$ the problem is called unbalanced transportation problem and solve it by adding slack variable to demand or supply with total cost zero.

Now, we have transportation problem on the m + n-١ of the basic variables and we can find the initial solution of the problem by using one the following methods:-

١) North west corner method.

٢) Least cost method.

٣) Vogel's approximation method.

Then we iterate optimal solution, we must apply one of the following methods:-

٤) Stepping stone method.

٥) Modified distribution method.

٢-١ Introduction

In real life there are many problems which deal with uncertainty in parameters; therefore the fuzzy set theory is uncertainly.

In the last few years, there is linked between fuzzy set theory and transportation problem to get a new subject which called by fuzzy transportation problem, the aim of this subject is to find the solution which have minimum transportation cost.

In this chapter, we deals with many algorithms which depend on different ranking function and suggested a new two algorithms which depend on new ranking function, then take two numerical examples to apply all these algorithms and compare between them out of total cost.

٢-٢ Fuzzy Transportation Problem

In the real life applications supply, demand and units transportation cost may be uncertain due to several factors.

Fuzzy transportation problem is a transportation problem whose decision variables are fuzzy numbers. **[٢٠]**

The fuzzy transportation problem is a special cases of fuzzy linear programming which fuzzified the constrains and objective function in linear programming through fuzzy numbers and used ranking function to obtain linear programming in the fuzzy sense. **[٨]**

The objective of the fuzzy transportation problem is to determine the shipping schedule that minimizes the total fuzzy transportation cost while satisfying the availability and requirement limits. **[٢٩]**

Mathematically, a fuzzy transportation problem can be stated in many cases which are as follows:-

١-The costs are fuzzy number

We can write this state as a linear programming problem which formulate as:-

$$\text{Minimize } z=\sum_{k=١}^{m}\sum_{l=١}^{n}\tilde{c}_{kl}\,x_{kl}$$

S.t

$$\sum_{l}^{n}x_{kl}=S_k \qquad k=١,\dots,m$$

$$\sum_{k}^{m}x_{kl}=D_l \qquad l=١,\dots,n$$

$$x_{kl}\geq ٠ \qquad k=١,\dots,m,\quad l=١,\dots,n$$

٢-The costs and supply are fuzzy numbers.

This state can be written mathematically as:-

$$\text{Minimize } z=\sum_{k=١}^{m}\sum_{l=١}^{n}\tilde{c}_{kl}\,x_{kl}$$

S.t

$$\sum_{l}^{n}x_{kl}=\tilde{S}_k \qquad k=١,\dots,m$$

$$\sum_{k}^{m}x_{kl}=D_l \qquad l=١,\dots,n$$

$$x_{kl}\geq ٠ \qquad k=١,\dots,m,\quad l=١,\dots,n$$

٣-The supply and demand are fuzzy numbers.

Mathematically we can write this state as follows:

$$\text{Minimize } z=\sum_{k=١}^{m}\sum_{l=١}^{n}c_{kl}\,x_{kl}$$

S.t

$$\sum_{l}^{n}x_{kl}=\tilde{S}_k \qquad k=١,\dots,m$$

$$\sum_{k}^{m}x_{kl}=\tilde{D}_l \qquad l=١,\dots,n$$

$$x_{kl}\geq ٠ \qquad k=١,\dots,m,\quad l=١,\dots n$$

٤- The cost and demand are fuzzy numbers.

For this state, we can write the transportation problem as linear programming problem as follows:

$$\text{Minimize } z=\sum_{k=١}^{m}\sum_{l=١}^{n}\tilde{c}_{kl}\,x_{kl}$$

S.t

$$\sum_{l}^{n} x_{kl} = S_k \qquad k = ١, \ldots, m$$

$$\sum_{k}^{m} x_{kl} = \tilde{D}_l \qquad l = ١, \ldots, n$$

$$x_{kl} \geq ٠ \qquad k=١,\ldots,m,\ l=١,\ldots,n$$

٥- The fully fuzzy numbers.

We can write the fully transportation problem as linear programming problem which formulate as follows:

$$\text{Minimize } z=\sum_{k=١}^{m}\sum_{l=١}^{n}\tilde{c}_{kl}\,x_{kl}$$

S.t

$$\sum_{l}^{n} x_{kl} = \tilde{S}_k \qquad k = ١, \ldots, m$$

$$\sum_{k}^{m} x_{kl} = \tilde{D}_l \qquad l = ١, \ldots, n$$

$$x_{kl} \geq ٠ \qquad k=١,\ldots,m,\ \ l=١,\ldots,n$$

٢-٣ Ranking Function

Ordering of fuzzy quantities is based on extracting various features from fuzzy set. This feature may be a center of gravity, on area under the membership function.

A particular fuzzy set ranking method extracts a specific feature form fuzzy sets, after then ranks fuzzy sets are based on the feature.

As a result, it's reasonable to expect that different ranking methods can produce different ranking orders for the same sample of fuzzy set. **[٢٨]**

In many applications, ranking of fuzzy number is an important component of decision process. **[١١]**

Now, defining ranking function $R: F(\mu) \rightarrow R$ which maps each fuzzy number into the real line, $F(\mu)$ represents the set of all triangular fuzzy numbers. **[٣٢]**

For any two fuzzy numbers $\widetilde{A} = (a, b, c)$ and $\widetilde{B} = (a_١, b_١, c_١)$ have in F(M), we have the following comparison:

١) $\widetilde{A} \leq \widetilde{B}$ iff $R(A) \leq R(B)$.

٢) $\widetilde{A} \geq \widetilde{B}$ iff $R(A) \geq R(B)$.

٣) $\widetilde{A} \approx \widetilde{B}$ iff $R(A) \approx R(B)$.

٤) $\widetilde{A} - \widetilde{B} = \tilde{٠}$ iff $R(A) - R(B) = ٠$.

A triangular fuzzy number $\widetilde{A} = (a, b, c)$ in $F(\mu)$ is said to be positive if $R(\widetilde{A}) > ٠$ and denoted by $\widetilde{A} > \tilde{٠}$. **[٢٤]**

٢-٤ The Algorithms for Ranking Function

Suppose the triangular fuzzy number is represented by $\widetilde{A} = (a, b, c)$

Where b is the medium, a is the left width and c is the right width and has its membership function

$$\mu_A(x) = \begin{cases} \frac{(x-a)}{(b-a)} & a \le x \le b \\ ١ & x = b \\ \frac{(c-x)}{(c-b)} & b \le x \le c \end{cases} \quad \dots (١)$$

Now by using $\alpha - \text{cut} \;\; \alpha \in [٠,١]$ we get

$$\alpha = \frac{(x-a)}{(b-a)} \qquad\qquad \alpha = \frac{(c-x)}{(c-b)}$$

$$x = a + \alpha(b-a) \qquad\qquad x = c - \alpha(c-b)$$

$$\widetilde{A}^l{}_{(\alpha)} = a + \alpha(b-a) \dots (٢) \qquad \widetilde{A}^u{}_{(\alpha)} = c - \alpha(c-b) \dots (٣)$$

Where $\widetilde{A}^l{}_{(\alpha)}$ is abounded left continuous non decreasing function over [٠, ١] and $\widetilde{A}^u{}_{(\alpha)}$ is a bounded left continuous non increasing function over [٠, ١], then presented for arbitrary fuzzy numbers be an ordered pair of function $[\widetilde{A}^l{}_{(\alpha)}, \; \widetilde{A}^u{}_{(\alpha)}]$ where $\widetilde{A}^l{}_{(\alpha)} \le \widetilde{A}^u{}_{(\alpha)}$ and

$$\widetilde{A}^l{}_{(\alpha)} = \inf\{x: \widetilde{A}_{(x)} \ge \alpha\} \qquad \widetilde{A}^u{}_{(\alpha)} = \sup\{x: \widetilde{A}_{(x)} \ge \alpha\}.$$

٢-٤-١ The First Algorithm [١١]

Basirzadeh at (٢٠١١) depended on the ranking function of Yager at (١٩٨١) which found ranking function for trapezoidal and triangular membership. the ranking function for this algorithm is as follows by using the membership (١) and then applied equation (٢) & (٣) we have:-

$$R\,(\widetilde{A}_{(x)}) = \frac{١}{٢}\int_{٠}^{١} [\widetilde{A}^l{}_{(\alpha)} + \widetilde{A}^u{}_{(\alpha)}]\, d\alpha.$$

$$R(\widetilde{A}_{(x)}) = \frac{١}{٢}\int_{٠}^{١}[a + \alpha(b - a) + c - \alpha(c - b)]d\alpha.$$

$$R(\widetilde{A}_{(x)}) = \frac{١}{٢}\int_{٠}^{١}[\alpha(٢b - a - c) + (a + c)]d\alpha.$$

$$R(\widetilde{A}_{(x)}) = \frac{١}{٢}\left[\frac{\alpha^{٢}}{٢}\Big|_{٠}^{١}(٢b\text{-}a\text{-}c)+\alpha\Big|_{٠}^{١}(a + c)\right].$$

$$R(\widetilde{A}_{(x)}) = \frac{١}{٢}\left[\frac{١}{٢}(٢b - a - c) + (a + c)\right].$$

$$R(\widetilde{A}_{(x)}) = \frac{١}{٤}(٢b - a - c) + \frac{١}{٢}(a + c).$$

$$R(\widetilde{A}_{(x)}) = \frac{٢b-a-c+٢a+٢c}{٤}.$$

$$R(\widetilde{A}_{(x)}) = \frac{١}{٤}[a + ٢b + c].$$

٢-٤-٢ The Second Algorithm [٢٢]

Nagoor Gani and Abbas at (٢٠١٣) used the idea of Chiang at (٢٠٠٥), they stated that the demand and supply are fuzzy numbers only, where w is the weight for $\widetilde{A}^{l}_{(\alpha)}$ and (١-w) is weight for $\widetilde{A}^{u}_{(\alpha)}$.

$$R(\widetilde{A}_{(x)}) = \frac{\int_{٠}^{١}\alpha[w\widetilde{A}^{l}_{(\alpha)}+(١-w)\widetilde{A}^{u}_{(\alpha)}]d\alpha}{\int_{٠}^{١}\alpha d\alpha}$$

$$R(\widetilde{A}_{(x)}) = \frac{\int_{٠}^{١}\alpha[w(a+\alpha(b-a))+(١-\omega)(c-\alpha(c-b))]d\alpha}{\int_{٠}^{١}\alpha d\alpha}$$

$$R(\widetilde{A}_{(x)}) = \frac{\int_{٠}^{١}\alpha[wa+w\alpha(b-a)+(١-w)c-(١-w)\alpha(c-b)]d\alpha}{\int_{٠}^{١}\alpha d\alpha}$$

$$R(\widetilde{A}_{(x)}) = \frac{\int_{٠}^{١}[\alpha wa+\alpha^{٢}wb-\alpha^{٢}wa+(١-w)\alpha c-(١-w)\alpha^{٢}c+(١-w)\alpha^{٢}b]d\alpha}{\int_{٠}^{١}\alpha d\alpha}$$

$$R(\widetilde{A}_{(x)}) = \frac{\left[\frac{\alpha^{٢}}{٢}wa+\frac{\alpha^{٣}}{٣}wb-\frac{\alpha^{٣}}{٣}wa+\frac{\alpha^{٢}}{٢}(١-w)c-\frac{\alpha^{٣}}{٣}(١-w)c+\frac{\alpha^{٣}}{٣}(١-w)b\right]\Big|_{٠}^{١}}{\frac{\alpha^{٢}}{٢}\Big|_{٠}^{١}}$$

$$R(\widetilde{A}_{(x)}) = \frac{\frac{٣wa+٢wb-٢wa+٣c-٣wc-٢c+٢wc+٢b-٢wb}{٦}}{\frac{١}{٢}}$$

$$R(\widetilde{A}_{(x)}) = \frac{١}{٣}[w(a-c)+٢b+c].$$

٢-٤-٣ The Third Algorithm [٢٤]

This algorithm proposed by Naresh Kumar and Ghuru at (٢٠١٤) by using symmetric triangular fuzzy number $\widetilde{A}_{(x)}$=(a, b ,c) if a=c then $\widetilde{A}_{(x)}$ is called symmetric fuzzy number.

The idea of this algorithm is based for every $\widetilde{A} \in F(\mu)$ where $F(\mu)$ is fuzzy real line on graded mean which formulate by using membership (*) and then applied equation (١) & (٢) as follows:

$$R(\widetilde{A}_{(x)}) = \frac{\frac{١}{٢}\int_{٠}^{١} \alpha[\widetilde{A}^{l}{}_{(\alpha)}+\widetilde{A}^{u}{}_{(\alpha)}]d\alpha}{\int_{٠}^{١} \alpha d\alpha}$$

$$R(\widetilde{A}_{(x)}) = \frac{\frac{١}{٢}\int_{٠}^{١} \alpha[a+\alpha(b-a)+c-\alpha(c-b)]d\alpha}{\int_{٠}^{١} \alpha d\alpha}$$

$$R(\widetilde{A}_{(x)}) = \frac{\frac{١}{٢}\int_{٠}^{١}[\alpha a+\alpha^{٢}b-\alpha^{٢}a+\alpha c-\alpha^{٢}c+\alpha^{٢}b]d\alpha}{\int_{٠}^{١} \alpha d\alpha}$$

$$R(\widetilde{A}_{(x)}) = \frac{\frac{١}{٢}\left[\frac{a}{٢}+\frac{b}{٣}-\frac{a}{٣}+\frac{c}{٢}-\frac{c}{٣}+\frac{b}{٣}\right]}{\frac{١}{٢}}$$

$$R(\widetilde{A}_{(x)}) = \frac{١}{٦}[a+٤b+c].$$

٢-٤-٤ Proposed Algorithm (١)

The idea of this algorithm depends on the idea for Nagoor Gani and Abbas at (٢٠١٣), they studied that the demand and supply are fuzzy numbers only, but in this algorithm we take fully fuzzy numbers.

Now, by using the following triangular membership:-

$$\mu_A(x) = \begin{cases} \frac{\lambda(x-a)}{(b-a)} & a \le x \le b \\ \lambda & x = b \\ \frac{\lambda(c-x)}{(c-b)} & b \le x \le c \end{cases} \quad \dots(٤)$$

By using α-cut, where $\alpha \in [٠,١]$ and $٠ \le \alpha \le \lambda$ and $٠ \le \lambda \le ١$, then

$$\alpha = \frac{\lambda(x-a)}{(b-a)} \qquad \alpha = \frac{\lambda(c-x)}{(c-b)}$$

$$x = a + \frac{\alpha}{\lambda}(b-a) \qquad x = c - \frac{\alpha}{\lambda}(c-b)$$

$$\widetilde{A}^{l}_{(\alpha)} = a + \frac{\alpha}{\lambda}(b-a) \ \dots(٥) \qquad \widetilde{A}^{u}_{(\alpha)} = c - \frac{\alpha}{\lambda}(c-b) \ \dots(٦)$$

Where $\widetilde{A}^{l}_{(\alpha)}$ is abounded left continuous non decreasing function over $[٠,\lambda]$ and $\widetilde{A}^{u}_{(\alpha)}$ is a bounded left continuous non increasing function over $[٠,\lambda]$, then presented for arbitrary fuzzy numbers be an ordered pair of function $[\widetilde{A}^{l}_{(\alpha)}, \ \widetilde{A}^{u}_{(\alpha)}]$ where $\widetilde{A}^{l}_{(\alpha)} \le \widetilde{A}^{u}_{(\alpha)}$, let w is weight for $\widetilde{A}^{l}_{(\alpha)}$ and (١-w) is weight for $\widetilde{A}^{u}_{(\alpha)}$.

$$R(\widetilde{A}_{(x)}) = \frac{\int_{٠}^{\lambda} \alpha^{٢} \, [w\,\widetilde{A}^{l}_{(\alpha)} + (١-w)\widetilde{A}^{u}_{(\alpha)}]d\alpha}{\int_{٠}^{\lambda} \alpha^{٢} d\alpha}$$

$$R(\widetilde{A}_{(x)}) = \frac{\int_{٠}^{\lambda} \alpha^{٢} \left[wa + \frac{w\alpha}{\lambda}(b-a) + (١-w)c - \frac{(١-w)\alpha}{\lambda}(c-b) \right] d\alpha}{\int_{٠}^{\lambda} \alpha^{٢} d\alpha}$$

$$R\,(\widetilde{A}_{(x)}) = \frac{\int_{٠}^{\lambda}[\alpha^{٢}wa+\frac{\alpha^{٣}w}{\lambda}(b-a)+\alpha^{٢}(١-w)c-\frac{\alpha^{٣}(١-w)}{\lambda}(c-b)]}{\int_{٠}^{\lambda}\alpha^{٢}d\alpha}$$

$$R\,(\widetilde{A}_{(x)}) = \frac{\frac{\lambda^{٣}wa}{٣}+\frac{\lambda^{٤}w(b-a)}{٤\lambda}+\frac{\lambda^{٣}(١-w)c}{٣}-\frac{\lambda^{٤}(١-w)(c-b)}{٤\lambda}}{\frac{\lambda^{٣}}{٣}}$$

$$R\,(\widetilde{A}_{(x)}) = \frac{\frac{\lambda^{٣}[٤wa+٣w(b-a)+٤(١-w)c-٣(١-w)(c-b)}{١٢}}{\frac{\lambda^{٣}}{٣}}$$

$$R\,(\widetilde{A}_{(x)}) = \frac{١}{٤}[w(a-c)+٣b+c].$$

٢-٤-٥ Proposed Algorithm (٢)

The idea of this algorithm depends for previous proposed algorithm on using the membership (٤) and applied equations (٥) & (٦) to obtain the following:-

$$R\,(\widetilde{A}_{(\alpha)}) = \frac{\left[\frac{١}{٢}\int_{٠}^{\lambda}\alpha^{٢}\,[\widetilde{A}^{l}{}_{(\alpha)}+\widetilde{A}^{u}{}_{(\alpha)}\,]d\alpha\right]}{\int_{٠}^{\lambda}\alpha^{٢}\,d\alpha}$$

$$R\,(\widetilde{A}_{(\alpha)}) = \frac{\left[\frac{١}{٢}\int_{٠}^{\lambda}\alpha^{٢}\,[a+\frac{\alpha}{\lambda}(b-a)+\,c-\frac{\alpha}{\lambda}\,(c-b)]d\alpha\right]}{\int_{٠}^{\lambda}\alpha^{٢}\,d\alpha}$$

$$R\,(\widetilde{A}_{(\alpha)}) = \frac{\left[\frac{١}{٢}\int_{٠}^{\lambda}[\alpha^{٢}a+\frac{\alpha^{٣}}{\lambda}(b-a)+\alpha^{٢}c-\frac{\alpha^{٣}}{\lambda}\,(c-b)]d\alpha\right]}{\int_{٠}^{\lambda}\alpha^{٢}\,d\alpha}$$

$$R\,(\widetilde{A}_{(\alpha)}) = \frac{\left[\frac{١}{٢}\left[\frac{\alpha^{٣}}{٣}a+\frac{\alpha^{٤}}{٤\lambda}(b-a)+\frac{\alpha^{٣}}{٣}c-\frac{\alpha^{٤}}{٤\lambda}\,(c-b)\right]\Big|_{٠}^{\lambda}\right]}{\frac{\alpha^{٣}}{٣}\,\Big|_{٠}^{\lambda}}$$

$$R\,(\widetilde{A}_{(\alpha)}) = \frac{\left[\frac{1}{2}\left[\frac{\lambda^{3}}{3}a+\frac{\lambda^{4}}{4\lambda}(b-a)+\frac{\lambda^{3}}{3}c-\frac{\lambda^{4}}{4\lambda}\,(c-b)\right]\right]}{\frac{\lambda^{3}}{3}}$$

$$R(\widetilde{A}_{(\alpha)}) = \frac{\left[\frac{١}{٢}\left[\frac{\lambda^{٣}}{٣}a+\frac{\lambda^{٣}}{٤}(b-a)+\frac{\lambda^{٣}}{٣}c-\frac{\lambda^{٤}}{٤}(c-b)\right]\right]}{\frac{\lambda^{٣}}{٣}}$$

$$R(\widetilde{A}_{(\alpha)}) = \frac{\frac{\lambda^{٣}}{٢}\left[\frac{a}{٣}+\frac{(b-a)}{٤}+\frac{c}{٣}-\frac{(c-b)}{٤}\right]}{\frac{\lambda^{٣}}{٣}}$$

$$R(\widetilde{A}_{(\alpha)}) = \frac{\frac{\lambda^{٣}}{٢}\left[\frac{٤a+٣b-٣a}{١٢}+\frac{٤c-٣c+٣b}{١٢}\right]}{\frac{\lambda^{٣}}{٣}}$$

$$R(\widetilde{A}_{(\alpha)}) = \frac{[\mathbf{a+6b+c}]}{\mathbf{8}}.$$

٢-٥ Numerical Examples [١]

In this section, we take two examples to the fuzzy transportation problems, the first example when the sum of supply and the sum of demand are equal ($\sum_{k=١}^{m} S_k = \sum_{l=١}^{n} D_l$), the second example when the sum of supply and sum of demand are unequal ($\sum_{k=١}^{m} S_k \neq \sum_{l=١}^{n} D_l$).

٢-٥-١ The First Example (٢.١)

A company contains three sources which are denoted by $S_١, S_٢, S_٣$ and three destinations which are denoted by $D_١, D_٢, D_٣$.

Table (٢-١) represents crisp transportation problem balanced

	$D_١$	$D_٢$	$D_٣$	Supply
$S_١$	١٠	٢	١٦	٢٤
$S_٢$	١	٨	٤	٢٨
$S_٣$	١٤	١٢	٦	٨

Demand	١٨	٢٠	٢٢	٦٠

Case ١:- Crisp Transportation problem

In the beginning, we solve the transportation problem by using Vogel's approximation algorithm with modified distribution algorithm

to get optimal solution.

Table (٢-٢) represents optimal solution for crisp problem

	$D_{١}$	$D_{٢}$	$D_{٣}$	Supply
$S_{١}$	١٠ ٤	٢ ٢٠	١٦	٢٤
$S_{٢}$	١ ١٤	٨	٤ ١٤	٢٨
$S_{٣}$	١٤	١٢	٦ ٨	٨
Demand	١٨	٢٠	٢٢	٦٠

T.c=٤٠+٤٠+١٤+٥٦+٤٨ =١٩٨ units.

Noting that, the source $S_{١}$ provides the first destination $D_{١}$ by (٤) units and second destination $D_{٢}$ by (٢٠) units. The source $S_{٢}$ provides the first destination by (١٤) units and third destination $D_{٣}$ by (١٤) units. The source $S_{٣}$ provides the destination $D_{٣}$ by (٨) units, then the total Cost is (١٩٨) units .

Case ٢:- Fuzzy Transportation Problem

In this case, we transform the crisp transportation problem to fuzzy transportation problem and solve by different ranking functions.

Let $\Delta_{١} = ٠.٥, \Delta_{٢} = ١$, then

$$\widetilde{A} = [(c_{kl} - \Delta_{١}, c_{kl}, c_{kl} + \Delta_{٢}), (S_k - \Delta_{١}, S_k, S_k + \Delta_{٢}), (D_l - \Delta_{١}, D_l, D_l + \Delta_{٢})].$$

Table (٢-٣) represents fuzzy transportation problem

	D١	D٢	D٣	Supply
S١	(٩.٥,١٠,١١)	(١.٥,٢,٣)	(١٥.٥,١٦,١٧)	(٢٣.٥ ,٢٤,٢٥)
S٢	(٠.٥,١,٢)	(٧.٥,٨,٩)	(٣.٥,٤,٥)	(٢٧.٥,٢٨,٢٩)
S٣	(١٣.٥,١٤,١٥)	(١١.٥,١٢,١٣)	(٥.٥,٦,٧)	(٧.٥,٨,٩)
Demand	(١٧.٥,١٨,١٩)	(١٩.٥,٢٠,٢١)	(٢١.٥,٢٢,٢٣)	(٥٨.٥,٦٠,٦٣)

∗ **First Algorithm**

$$R\,(\tilde{A}_{(x)}) = \frac{١}{٤}\,[a + ٢b + c].$$

Table (٢-٤) represents the optimal solution of first algorithm

	$D_{١}$	$D_{٢}$	$D_{٣}$	Supply
$S_{١}$	١٠.١٢٥ ٤	٢.١٢٥ ٢٠.١٢٥	١٦.١٢٥	٢٤.١٢٥
$S_{٢}$	١.١٢٥ ١٤.١٢٥	٨.١٢٥	٤.١٢٥ ١٤	٢٨.١٢٥
$S_{٣}$	١٤.١٢٥	١٢.١٢٥	٦.١٢٥ ٨.١٢٥	٨.١٢٥
Demand	١٨.١٢٥	٢٠.١٢٥	٢٢.١٢٥	٦٠.٣٧٥

T.c= ٤٠.٥٠+٤٢.٧٧+١٥.٨٩+٥٧.٧٥+٤٩.٧٧

=٢٠٦.٦٨ units.

Noting that, the source $S_{١}$ provides the first destination $D_{١}$ by (٤) units and second destination $D_{٢}$ by (٢٠.١٢٥) units. The source $S_{٢}$ provides the first destination by (١٤.١٢٥) units and third destination $D_{٣}$ by (١٤) units. The source $S_{٣}$ provides the destination $D_{٣}$ by (٨.١٢٥) units, then the total cost is (٢٠٦.٦٨) units.

* **Second Algorithm**

$$R\,(\widetilde{A}_{(x)}) = \frac{١}{٣}[w(a - c) + ٢b + c], w \in [٠,١]$$

Accordingly, when $w \in [٠,١]$ we sub state all the values of w to get the solution which scheduling in the following table:-

Table (٢-٥) represents the optimal solution of second algorithm

Weight	Total cost
٠	٢٢١.١٠
٠.١	٢١٧.٧٧
٠.٢	٢١٤.٢٤
٠.٣	٢١٠.٧٣
٠.٤	٢٠٧.٢٣
٠.٥	٢٠٣.٧٥
٠.٦	٢٠٠.٢٨
٠.٧	١٩٦.٨٣
٠.٨	١٩٣.٣٩
٠.٩	١٨٦.٧٦
١	١٨٦.٧٦

The best weigh which give us the minimum transportation costs are w≥ ٠.٧.

∗ **Third algorithm**

$$R\ (\widetilde{A}_{(x)}) = \frac{١}{٦}[a + ٤b + c].$$

Table (٢-٦) represents the optimal solution of third algorithm

	$D_{١}$	$D_{٢}$	$D_{٣}$	Supply
$S_{١}$	١٠.٠٨٣ ٤	٢.٠٨٣ ٢٠.٠٨٣	١٦.٠٨٣	٢٤.٠٨٣
$S_{٢}$	١.٠٨٣ ١٤.٠٨٣	٨.٠٨٣	٤.٠٨٣ ١٤	٢٨.٠٨٣
$S_{٣}$	١٤.٠٨٣	١٢.٠٨٣	٦.٠٨٣ ٨.٠٨٣	٨.٠٨٣
Demand	١٨.٠٨٣	٢٠.٠٨٣	٢٢.٠٨٣	٦٠.٢٤٩

T.c=٤٠.٣٣+٤١.٨٣+١٥.٢٥+٥٧.١٦+٤٩.١٧

=٢٠٣.٧٥ units.

Noting that, the source $S_{١}$ provides the first destination $D_{١}$ by (٤) units and second destination $D_{٢}$ by (٢٠.٠٨٣) units. The source $S_{٢}$ provides the first destination by (١٤.٠٨٣) units and third destination $D_{٣}$ by (١٤) units. The source $S_{٣}$ provides the destination $D_{٣}$ by (٨.٠٨٣) units, then the total cost is (٢٠٣.٧٥) units.

∗ **Proposed Algorithm (١)**

$$R(\widetilde{A}_{(x)}) = \frac{١}{٤}[w(a - c) + ٣b + c].$$

Accordingly, when w ∈ [٠,١] we sub state all the values of w to get the solution which scheduling in the following table:-

Table (٢-٧) represents the optimal solution of proposed algorithm (١)

Weight	Total cost
٠	٢١٥.٤٤
٠.١	٢١٢.٧٦
٠.٢	٢١٠.١٧
٠.٣	٢٠٧.٥١
٠.٤	٢٠٤.٩٣
٠.٥	٢٠٢.٢٩
٠.٦	١٩٩.٧٣
٠.٧	١٩٧.٥٦
٠.٨	١٩٤.٥٦
٠.٩	١٩١.٩٥
١	١٨٩.٤٢

The best weight which give us the minimum transportation costs are w≥ ٠.٧.

∗ **Proposed Algorithm (٢)**

$$R\,(\widetilde{A}_{(x)}) = \frac{١}{٨}\,[a + ٦b + c].$$

Table (٢-٨) represents the optimal solution of proposed algorithm (٢)

	$D_١$	$D_٢$	$D_٣$	Supply
$S_١$	١٠.٠٦٢٥ ٤	٢.٠٦٢٥ ٢٠.٠٦٢٥	١٦.٠٦٢٥	٢٤.٠٦٢٥
$S_٢$	١.٠٦٢٥ ١٤.٠٦٢٥	٨.٠٦٢٥	٤.٠٦٢٥ ١٤	٢٨.٠٦٢٥
$S_٣$	١٤.٠٦٢٥	١٢.٠٦٢٥	٦.٠٦٢٥ ٨.٠٦٢٥	٨.٠٦٢٥
Demand	١٨.٠٦٢٥	٢٠.٠٦٢٥	٢٢.٠٦٢٥	٦٠.١٨٧٥

T.c= ٤٠.٢٥+٤١.٣٨+١٤.٩٤+٥٦.٨٨+٤٨.٨٨

=٢٠٢.٣٢ units.

Noting that, the source $S_١$ provides the first destination $D_١$ by (٤) units and second destination $D_٢$ by (٢٠.٠٦٢٥) units. The source $S_٢$ provides the first destination by (١٤.٠٦٢٥) units and third destination $D_٣$ by (١٤) units. The source $S_٣$ provides the destination $D_٣$ by (٨.٠٦٢٥) units, then the total cost is (٢٠٢.٣٢) units.

٢-٥-٢ Second Example (٢.٢)

One of factory that produces fertilizer possesses chemical manufacturers in different locations. The energy productivity of these plants is (١٤٠), (١٢٠) a bag the first day of the plant and the second, respectively.

These factories are processing ٣ markets center big is ($S_١, S_٢, S_٣$) as needed(١٥٠) ,(١٧٠), (٨٠) a bag per day of fertilizer, the cost of transportation of the first factory to catalog the center first, second and third are (١.٧),(١),(١.٤) dinars,respectively, and the cost of transportation from the second factory to catalog the centers first, second and third are (١.٥), (١.١),(١.٣) dinars respectively. The administration at this property you want to transport bags of fertilizer from their manufacturers to the three market centers the bags to a achieve lower possible cost

Table (٢-٩) represents unbalanced transportation problem

	$D_١$	$D_٢$	$D_٣$	Supply
$S_١$	١.٧	١	١.٤	١٤٠
$S_٢$	١.٥	١.١	١.٣	١٢٠
Demand	١٥٠	١٧٠	٨٠	٢٦٠ / ٤٠٠

Considered that, $\sum_{k=١}^{٢} S_k = ٢٦٠$, $\sum_{l=١}^{٣} D_l = ٤٠٠$

$\sum_{k=١}^{٢} S_k \neq \sum_{l=١}^{٣} D_l$, then we add slack variable to the supply with zero cost.

Taking $\sum_{k=١}^{٢} S_k - \sum_{l=١}^{٣} D_l = ١٤٠$

Table (٢-١٠) represents balanced transportation problem

	$D_١$	$D_٢$	$D_٣$	Supply
$S_١$	١.٧	١	١.٤	١٤٠
$S_٢$	١.٥	١.١	١.٣	١٢٠
$S_٣$	٠	٠	٠	١٤٠
Demand	١٥٠	١٧٠	٨٠	٤٠٠

Case ١:- Crisp Transportation Problem

In the beginning, we solve the transportation problem by using Vogel's approximation algorithm with modified distribution algorithm and (Win QSP) to get the optimal solution.

	$D_١$	$D_٢$	$D_٣$	Supply
$S_١$	١.٧	١ ١٤٠	١.٤	١٤٠
$S_٢$	١.٥ ١٠	١.١ ٣٠	١.٣ ٨٠	١٢٠
$S_٣$	٠ ١٤٠	٠	٠	١٤٠

Demand	١٥٠	١٧٠	٨٠	٤٠٠

Table (٢-١١) represents the optimal solution for crisp problem

T.c=١٥+١٤٠+٣٣+١٠٤ =٢٩٢ units.

Noting that, the source $S_١$ provisde the second destination $D_٢$ by (١٤٠) units. and the source $S_٢$ provides the first destination $D_١$ by (١٠) units and two destinations $D_٢$ by (٣٠) units and third destination $D_٣$ by (٨٠) units. The source $S_٣$ provides the destination $D_١$ by (١٤٠) units, then the total cost is (٢٩٢) units.

Case ٢:- Fuzzy Transportation Problem

In this case, we transform the crisp transportation problem to fuzzy transportation problem and then solve this problem by using different ranking functions. Let $\Delta_١ = ٠.٧$, $\Delta_٢ = ١.٤$, then

$$\widetilde{A} = [(c_{kl} - \Delta_١, c_{kl}, c_{kl} + \Delta_٢), (S_k - \Delta_١ S_k, S_k + \Delta_٢), (D_l - \Delta_١, D_l, D_l + \Delta_٢)]$$

Table (٢-١٢) represents fuzzy transportation problem

	$D_١$	$D_٢$	$D_٣$	Supply
$S_١$	(١,١.٧,٣.١)	(٠.٣,١,٢.٤)	(٠.٧,١.٤,٢.٨)	(١٣٩.٣,١٤٠,١٤١.٤
$S_٢$	(٠.٨,١.٥,٢.٩)	(٠.٤,١.١,٢.٥)	(٠.٦,١.٣,٢.٧)	(١١٩.٣,١٢٠,١٢١.٤
$S_٣$	(-٠.٧,٠,١.٤)	(-٠.٧,٠,١.٤)	(-٠.٧,٠,١.٤)	(١٣٩.٣,١٤٠,١٤١.٤
Demand	(١٤٩.٣,١٥٠,١٥١.٤)	(١٦٩.٣,١٧٠,١٧١.٤)	(٧٩.٣,٨٠,٨١.٤)	(٣٩٧.٩,٤٠٠,٤٠٤.٢

* **First algorithm**

$$R(\tilde{A}) = \frac{١}{٤}[a+٢b+c].$$

Table (٢-١٣) represents the optimal solution of first algorithm

	$D_١$	$D_٢$	$D_٣$	Supply
$S_١$	١.٨٧٥	١.١٧٥ ١٤٠.١٧٥	١.٥٧٥	١٤٠.١٧٥
$S_٢$	١.٦٧٥ ١٠	١.٢٧٥ ٣٠	١.٤٧٥ ٨٠.١٧٥	١٢٠.١٧٥
$S_٣$	٠.١٧٥ ١٤٠.١٧٥	٠.١٧٥	٠.١٧٥	١٤٠.١٧٥
Demand	١٥٠.١٧٥	١٧٠.١٧٥	٨٠.١٧٥	٤٠٠.٥٢٥

T.c=١٦٤.٧١+١٦.٧٥+٣٨.٢٥+١١٨.٢٦+٢٤.٥٣

=٣٦٢.٥٠ units.

Noting that, the source $S_١$ provides the second destination $D_٢$ by (١٤٠.١٧٥) units. and the source $S_٢$ provides the first destination $D_١$ by (١٠) units and two destinations $D_٢$ by (٣٠) units and third destination $D_٣$ by (٨٠.١٧٥) units. The source $S_٣$ provides the destination $D_١$ by (١٤٠.١٧٥) units, then the total cost is(٣٦٢.٤) units.

∗ **Second algorithm**

$$R(\widetilde{A}_{(x)}) = \frac{١}{٣}[w(a-c) + ٢b + c], w \in [٠,١]$$

Accordingly, when $w \in [٠,١]$ we sub state all the values of w to get the solution which a scheduled in the following table:-

Table (٢-١٤) represents the optimal solution of second algorithm

Weight	Total cost
٠	٤٨٠.١٢
٠.١	٤٥١.٧٨
٠.٢	٤٠٥.٢١
٠.٣	٣٧٧.٦٣
٠.٤	٣٦٩.٩٣
٠.٥	٣٣٨.٧١
٠.٦	٣١٠.٥١
٠.٧	٢٨٢.٤٩
٠.٨	٢٤٠.٣٦
٠.٩	٢٢٦.٢٤
١	١٩٨.١٦٥

The best weight which give us the minimum transportation costs are w≥ ٠.٧.

∗ **Third algorithm**

$$R\,(\tilde{A}_{(x)}) = \frac{١}{٦}\,[a + ٤b + c].$$

Table (٢-١٥) represents the optimal solution for third algorithm

	$D_{١}$	$D_{٢}$	$D_{٣}$	Supply
$S_{١}$	١.٨١٦	١.١١٦ ١٤٠.١١٦	١.٥١٦	١٤٠.١١٦
$S_{٢}$	١.٦١٦ ١٠	١.٢١٦ ٣٠	١.٤١٦ ٨٠.١١٦	١٢٠.١١٦
$S_{٣}$	٠.١١٦ ١٤٠.١١٦	٠.١١٦	٠.١١٦	١٤٠.١١٦
Demand	١٥٠.١١٦	١٧٠.١١٦	٨٠.١١٦	٤٠٠.٣٥

T.c=١٦.١٦+١٦.٢٥+١٥٦.٣٧+٣٦.٥+١١٣.٤٤

=٣٣٨.٧١ units.

Noting that, the source $S_{١}$ provides the second destination $D_{٢}$ by (١٤٠.١١٦) units. and The source $S_{٢}$ provides the first destination $D_{١}$ by (١٠) units and two destination $D_{٢}$ by (٣٠) units and third destination $D_{٣}$ by (٨٠.١٦٦) units. The source $S_{٣}$ provides the destination $D_{١}$ by (١٤٠.١٦٦) units, then the total cost is (٣٣٨.٧١) units.

∗ **Proposed Algorithm (١)**

$$R(\widetilde{A}) = \frac{1}{4}[w(a - c) + ٣b + c], w \in [٠,١]$$

Table (٢-١٦) represents the optimal solution for proposed algorithm (١)

Weight	Total cost
٠	٤٣٣.١٧
٠.١	٣٩٤.٦٣
٠.٢	٣٩٠.٧٤
٠.٣	٣٦٩.٣٥
٠.٤	٣٤٨.٣٨
٠.٥	٣٢٧.٠٢
٠.٦	٣٠٦.٠٨
٠.٧	٢٨٤.٩٠
٠.٨	٢٦٣.٨٥
٠.٩	٢٤٢.٧٠
١	٢٢٠.٩٥

The best weight which give us the minimum transportation costs are w≥ ٠.٧.

∗ **Proposed Algorithm (٢)**

$$R\ (\widetilde{A}) = \frac{\mathbf{1}}{\mathbf{8}}[a + ٦b + c]$$

Table (٢-١٧) represents the optimal solution for proposed algorithm (٢)

	D١	D٢	D٣	Supply
S١	١.٧٨٧٥	١.٠٨٧٥ ١٤٠.٠٨٧٥	١.٤٨٧٥	١٤٠.٠٨٧٥
S٢	١.٥٨٧٥ ١٠	١.١٨٧٥ ٣٠	١.٣٨٧٥ ٨٠.٠٨٧٥	١٢٠.٠٨٧٥
S٣	٠.٠٨٧٥ ١٤٠.٠٨٧٥	٠.٠٨٧٥	٠.٠٨٧٥	١٤٠.٠٨٧٥
Demand	١٥٠.٠٨٧٥	١٧٠.٠٨٧٥	٨٠.٠٨٧٥	٤٠٠.٢٦٢٥

T.c=١٥٢.٣٥+١٥.٨٨+٣٥.٦٣+١١١.١٢+١٢.٢٦

=٣٢٧.٢٢ units.

Noting that, the source S١ provides the second destination D٢ by (`١٤٠.٠٨٧٥) units. and The source S٢ provides the first destination D١ by (١٠) units and two destinations D٢ by (٣٠) units and third destination D٣ by (٨٠..٠٨٧٥) units. The source S٣ provides the destination D١ by (١٤٠.٠٨٧٥) unit, then the total cost is (٣٢٧.٢٢) units.

٢-٦ Compare between algorithms

Algorithms	First example	Second example
Crisp	١٩٨	٢٩٢
First	٢٠٦.٦٨	٣٦٢.٤٩
Second	١٨٦.٧٦	٢٢٦.٢٤
Third	٢٠٣.٧٥	٣٣٨.٧١

In this section, we compare the total cost for all algorithms which depends on different ranking functions, also the proposed algorithm and crisp algorithm.

Table (٢-١٨) represents the comparison between all results algorithms

Proposed (١)	١٩١.٩٥	٢٤٢.٧٠
Proposed (٢)	٢٠٢.٣٢	٣٢٧.٢٢

Noting in the first example, the minimum total cost in the second algorithm and the proposed (١) algorithm near to the second algorithm.

In the second example, the minimum total cost also in second algorithm and the proposed (١) it's near to second algorithm. Then the best algorithms are second and proposed (١).

٣-١ Introduction

In this chapter we studied the ways in which in the second chapter on especially real data transfer and distribution of petroleum products from the warehouse to the governorates to get a cost less transfer was supply and demand and cost of transport coefficients fuzzy.

This chapter consists of (٣-٢) brief on Oil Products Distribution Company and the data obtained. In section (٣-٣) was the data extraction solution and results through the program (WinQSB). In section (٣-٤) we organized a comparison table between the results through the transport cost.

٣-٢ Brief summary about the company and the data

Review of the Ministry of Oil / Department of Studies and Planning, located in Philistine Street -ha channel in order to get transportation cost, demand and supply of petroleum products.

This department transform us to the distribution of petroleum products company located in Dura; in contracts division were obtained data for the transportation cost and noting that the transportation is by tanker trucks by contract with civil companies and that these company are (Rawasi company - Eyes company - Zubair Corporation - the flame company - wind carpet Company - Growing company - New Line Company - Iraq Jawwal company - the treasures of the earth company - company Ghadeer - Zahra Company - Anwar Karbala company - the fields of Lights company - mandate of the company Ali - Dar El Oyoun company - a smooth gloss gold company - Maali Burj company - the tender fields of company).

Told me by manager of department of contracts the governorates (Nasiriyah–Amara - Basra) where transportation is by especially cars to the

Ministry of oil / Distribution of petroleum products company / or the section on company cars and agents government stations or by the pipeline transportation and is not contracted with any civil companies that transport cost which zero.

As told me the governorates (Dohuk - Sulaymaniyah - Arbil) there is no data for transportation because there is all done within the region so it was exception.

Because of security in (Diyala, Anbar - Salah Al dien – Mosul) no complete data for transportation also have been exception. Also review equipped organizations in the distribution company were obtained data with actual sales of petroleum products and the amount of inventories in warehouses by the assets Division. It has been inform me by the manager that the increase and decrease in the quantities of demand and supply is increased by ٥٪ and ٤٪ for transportation cost.

Now, we will summarize the data obtained in the following tables and that these data are for ٢٠١٤ and for the governorates which are processed only from Basra Governorate.

Table (٣-١) represents data transport cost from warehouses to governorates

To / Form	Baghdad	Wasit	Karbala	Najaf	Diwaniya	Muthanna
Khor- Al Zubair	٦٢٠٠٠	٢٥٠٠٠	٣٢٠٠٠	٢٧٠٠٠	٢٣٠٠٠	٢١٠٠٠
Shuaiba	٦٠٠٠٠	٢٠٠٠٠	٣٠٠٠٠	٢٥٠٠٠	٢١٠٠٠	٢٠٠٠٠

Table (٣-٢) represents the amount of data to governorates demand for petroleum products m٣ / day

governorates	Baghdad	Wasit	Karbala	Najaf	Diwaniya	Muthanna
Demand	١٠٢٤٨.٨	١٦٤٤.١٨	١٦٨٧.٤	١٨٨٢.٢٢	١٤٦٥.٢١	٨٤٥.٠٩

Table (٣-٣) represents the amount of data to supply in the warehouses of oil products m٣ / day

Warehouses	Supply
Khor- Al Zubair	٧٣١٨.٢٥
Shuaiba	٦٦٠٧.١٤

٣-٣ Solving The Real data

In this section we solve the transportation problem by using (Win QSB).

Table (٣-٤) represents special data petroleum products (White oil, Gasoline and Gasoil)

From \ To	Baghdad	Wasit	Karbala	Najaf	Diwaniya	Muthanna	Supply
Khor- Al Zubair	٦٢٠٠٠	٢٥٠٠٠	٣٢٠٠٠	٢٧٠٠٠	٢٣٠٠٠	٢١٠٠٠	٧٣١٨.٢٥
Shuaiba	٦٠٠٠٠	٢٠٠٠٠	٣٠٠٠٠	٢٥٠٠٠	٢١٠٠٠	٢٠٠٠٠	٦٦٠٧.١٤
Demand	١٠٢٤٨.٨	١٦٤٤.١٨	١٦٨٧.٤	١٨٨٢.٢٢	١٤٦٥.٢١	٨٤٥.٠٩	١٣٩٢٥.٣٩ / ١٧٧٧٢.٩

Considered that, $\sum_{k=١}^{٢} S_k =$١٣٩٢٥.٣٩ , $\sum_{l=١}^{٣} D_l =$١٧٧٧٢.٩

$\sum_{k=١}^{٢} S_k \neq \sum_{l=١}^{٣} D_l$, then we adding slack variable to the supply with zero cost,

and taking $\sum_{k=١}^{٢} S_k - \sum_{l=١}^{٣} D_l =$٣٨٤٧.٥١

Table (٣-٥) represents a special data petroleum products the balanced (White oil, Gasoline and Gasoil)

TO / From	Baghdad	Wasit	Karbala	Najaf	Diwaniya	Muthanna	Supply
Khor- Al Zubair	٦٢٠٠٠	٢٣٠٠٠	٣٢٠٠٠	٢٧٠٠٠	٢٣٠٠٠	٢١٠٠٠	٧٣١٨.٢٥
Shuaiba	٦٠٠٠٠	٢٠٠٠٠	٣٠٠٠٠	٢٥٠٠٠	٢١٠٠٠	٢٠٠٠٠	٦٦٠٧.١٤
Warehouse	٠	٠	٠	٠	٠	٠	٣٨٤٧.٥١
Demand	١٠٢٤٨.٨	١٦٤٤.١٨	١٦٨٧.٤	١٨٨٢.٢٢	١٤٦٥.٢١	٨٤٥.٠٩	١٧٧٧٢.٩ / ١٧٧٧٢.٩

Case ١:- Crisp transportation problem

In the beginning, we solve the transportation problem by using (Win QSP) to get the optimal solution.

Table (٣-٦) represents optimal solution for real data

TO / From	Baghdad	Wasit	Karbala	Najaf	Diwaniya	Muthanna	Supply
Khor- Al Zubair	٦٢٠٠٠ ١٤٣٨.٣٣٠	٢٣٠٠٠	٣٢٠٠٠ ١٦٨٧.٤٠٠	٢٧٠٠٠ ١٨٨٢.٢٢٠	٢٣٠٠٠ ١٤٦٥.٢١٠	٢١٠٠٠ ٨٤٥.٢١	٧٣١٨.٢٥
Shuaiba	٦٠٠٠٠ ٤٩٦٢.٩٦٠	٢٠٠٠٠ ١٦٤٤.١٨	٣٠٠٠٠	٢٥٠٠٠	٢١٠٠٠	٢٠٠٠٠	٦٦٠٧.١٤
Warehouse	٠ ٣٨٤٧.٥١	٠	٠	٠	٠	٠	٣٨٤٧.٥١
Demand	١٠٢٤٨.٨	١٦٤٤.١٨	١٦٨٧.٤	١٨٨٢.٢٢	١٤٦٥.٢١	٨٤٥.٠٩	١٧٧٧٢.٩ ١٧٧٧٢.٩

The total cost = ٥٧٦١٠٣٦٤٠ units.

Case ٢:- Fuzzy transportation

In this case we transform the crisp transportation problem to fuzzy transportation problem and then solving by using different ranking function where $\widetilde{A} = [(c_{kl} - \Delta_{١}, c_{kl}, c_{kl} + \Delta_{٢}), (S_k - \Delta_{١}, S_k, S_k + \Delta_{٢}), (D_l - \Delta_{١}, D_l, D_l + \Delta_{٢})]$.

Table (٣-٧) represents fuzzy real data

TO / From	Baghdad	Wasit	Karbala	Najaf	Diwaniya	Muthanna	Supply
Khor- Al Zubair	(٥٩٥٢٠,٦٢٠٠٠, ٦٤٤٨٠)	(٢٢٠٨٠,٢٣٠٠٠, ٢٣٩٢٠)	(٣٠٧٢٠,٣٢٠٠٠, ٣٣٢٨٠)	(٢٥٩٢٠,٢٧٠٠٠, ٢٨٠٨٠)	(٢٢٠٨٠,٢٣٠٠٠, ٢٣٩٢٠)	(٢٠١٦٠,٢١٠٠٠,) ٢١٨٤٠	(٦٩٢.٣٣٧٥, ٧٣١٨.٢٥, ٧٦٨٤.١٦٢٥)
Shuaiba	(٥٧٦٠٠,٦٠٠٠٠, ٦٢٤٠٠)	(١٩٢٠٠,٢٠٠٠٠, ٢٠٨٠٠)	(٢٨٨٠٠,٣٠٠٠٠, ٣١٢٠٠)	(٢٤٠٠٠,٢٥٠٠٠, ٢٦٠٠٠)	(٢٠١٦٠,٢٣٠٠٠, ٢١٨٤٠٠)	(١٩٢٠٠,٢٠٠٠٠, ٢٠٨٠٠)	(٦٢٧٦.٧٨٣, ٦٦٠٧.١٤, ٦٩٣٧.٤٩٧)
Warehouse	(٠,٠,٠)	(٠,٠,٠)	(٠,٠,٠)	(٠,٠,٠)	(٠,٠,٠)	(٠,٠,٠)	(٣٦٥٥.١٣٤٥, ٣٨٤٧.٥١, ٤٠٣٩.٨٨٥٥)
Demand	(٩٧٣٦.٣٦,١٠٢٤٨.٨ ,١٠٧٦١.٢٤)	(١٥٦١.٩٧١,١٦٤٤.١٨ ١٧٢٦.٣٨٩)	(١٦٠٣.٠٥,١٦٨٧.٤ , ١٧٧١.٧٥)	(١٧٨٨.١٠٩,١٨٨٢.٢٢, ١٩٧٦.٣٣١)	(١٣٩١.٩٤٩٥,١٤٦٥.٢١, ١٥٣٨.٤٧٠٥)	(٨٠٢.٨٣٥٥,٨٤٥.٠٩, ٨٨٧.٣٤٤٥)	

* **First Algorithm**

$$R(\tilde{A}) = \frac{١}{٤}[a+٢b+c].$$

Table (٣-٨) represents optimal solution for first algorithm of real data

TO / From	Baghdad	Wasit	Karbala	Najaf	Diwaniya	Muthanna	Supply
Khor- Al Zubair	٦٢٠٠٠ ١٤٣٨.٣٣٠	٢٣٠٠٠	٣٢٠٠٠ ١٦٨٧.٤٠٠	٢٧٠٠٠ ١٨٨٢.٢٢٠	٢٣٠٠٠ ١٤٦٥.٢١٠	٢١٠٠٠ ٨٤٥.٢١	٧٣١٨.٢٥
Shuaiba	٦٠٠٠٠ ٤٩٦٢.٩٦٠	٢٠٠٠٠ ١٦٤٤.١٨	٣٠٠٠٠	٢٥٠٠٠	٢١٠٠٠	٢٠٠٠٠	٦٦٠٧.١٤
Warehouse	٠ ٣٨٤٧.٥١	٠	٠	٠	٠	٠	٣٨٤٧.٥١
Demand	١٠٢٤٨.٨	١٦٤٤.١٨	١٦٨٧.٤	١٨٨٢.٢٢	١٤٦٥.٢١	٨٤٥.٠٩	١٧٧٧٢.٩ ١٧٧٧٢.٩

The total cost = ٥٧٦١٠٣٦٤٠ units.

∗ **Second Algorithm**

$R(\widetilde{A}) = \frac{١}{٣}[w(a-c)+٢b+c].$

Accordingly, when w ∈ [٠,١] we sub state all the values of w to get the solution which scheduling in the following table:-

Table (٣-٩) represents the optimal solution of second algorithm for all Weight

W	٠	٠.١	٠.٢	٠.٣	٠.٤	٠.٥	٠.٦	٠.٧	٠.٨	٠.٩	١
T.c	٥٤٢٦٩٠٤٠٠	٥٩٠٠٠٩٤٠٠	٥٨٦٥١٧٠٠	٥٨٣٠٣٤٧٠٠	٥٧٩٥٦٢٨٠٠	٥٧٦١٠١١٠٠	٥٧٢٦٤٩٦٠٠	٥٦٧٥٩٩٨٠٠	٥٦٥٦٤٩٣٠٠	٥٦٢٤٠٦٥٠٠	٥٥٨٩٤٦٠٠

The beast for this algorithm is [٠.٧, ١].

∗ **Third Algorithm**

$$R(\tilde{A}) = \frac{١}{٦}[a + ٤b + c].$$

Table (٣-١٠) represents represents optimal solution

From \ TO	Baghdad	Wasit	Karbala	Najaf	Diwaniya	Muthanna	Supply
Khor- Al Zubair	٦٢٠٠٠ ١٤٣٨.٣٣٠	٢٣٠٠٠	٣٢٠٠٠ ١٦٨٧.٤٠٠	٢٧٠٠٠ ١٨٨٢.٢٢٠	٢٣٠٠٠ ١٤٦٥.٢١٠	٢١٠٠٠ ٨٤٥.٢١	٧٣١٨.٢٥
Shuaiba	٦٠٠٠٠ ٤٩٦٢.٩٦٠	٢٠٠٠٠ ١٦٤٤.١٨	٣٠٠٠٠	٢٥٠٠٠	٢١٠٠٠	٢٠٠٠٠	٦٦٠٧.١٤
Warehouse	٠ ٣٨٤٧.٥١	٠	٠	٠	٠	٠	٣٨٤٧.٥١
Demand	١٠٢٤٨.٨	١٦٤٤.١٨	١٦٨٧.٤	١٨٨٢.٢٢	١٤٦٥.٢١	٨٤٥.٠٩	١٧٧٧٢.٩ \ ١٧٧٧٢.٩

The total cost = ٥٧٦١٠٣٦٤٠ units.

∗ **Proposed Algorithm (١)**

$$R(\widetilde{A}) = \frac{١}{٤}[w(a-c)+٣b+c], w \in [٠,١]$$

Table (٣-١١) represents the optimal solution for all Weight for proposed algorithm (١)

W	٠	٠.١	٠.٢	٠.٣	٠.٤	٠.٥	٠.٦	٠.٧	٠.٨	٠.٩	١
T.c	٥٨٩١٣٥٤٠٠	٥٨٦٥١٧١٠٠	٥٨٣٩٠٤٤٠٠	٥٨١٢٩٧٥٠٠	٥٧٨٦٩٦٤٠٠	٥٧٦١٠١١٠٠	٥٧٣٥٠٤٨٠٠	٥٧٠٩٢٧٧٠٠	٥٦٨٢٦٥٦٠٠	٥٦٥٧٧٧٣٠٠	٥٦٣٢١٠٩٠٠

Note from the table above there are inverse relationship between weight and costs, and that the greater the weight the small costs.

∗ **Proposed algorithm (٢)**

$$R\,(\widetilde{A}) = \frac{١}{٨}\,[a + ٦b + c].$$

Table (٣-١٢) represents optimal solution for proposed algorithm (٢)

TO / From	Baghdad	Wasit	Karbala	Najaf	Diwaniya	Muthanna	Supply
Khor- Al Zubair	٦٢٠٠٠ ١٤٣٨.٣٣٠	٢٣٠٠٠	٣٢٠٠٠ ١٦٨٧.٤٠٠	٢٧٠٠٠ ١٨٨٢.٢٢٠	٢٣٠٠٠ ١٤٦٥.٢١٠	٢١٠٠٠ ٨٤٥.٢١	٧٣١٨.٢٥
Shuaiba	٦٠٠٠٠ ٤٩٦٢.٩٦٠	٢٠٠٠٠ ١٦٤٤.١٨	٣٠٠٠٠	٢٥٠٠٠	٢١٠٠٠	٢٠٠٠٠	٦٦٠٧.١٤
Warehouse	٠ ٣٨٤٧.٥١	٠	٠	٠	٠	٠	٣٨٤٧.٥١
Demand	١٠٢٤٨.٨	١٦٤٤.١٨	١٦٨٧.٤	١٨٨٢.٢٢	١٤٦٥.٢١	٨٤٥.٠٩	١٧٧٧٢.٩ ١٧٧٧٢.٩

The Total cost = ٥٧٦١٠٣٦٤٠ units.

٣-٤ Comparison

In this section we organized a comparison between the crisp transportation and all methods ranking function through the transport cost to the results of real data.

Table (٣-١٣) represents comparison between all results algorithms

Algorithms	Optimal solution
Crisp	٥٧٦١٠٣٦٤٠
First	٥٧٦١٠٣٦٤٠
Second	٥٦٢٤٠٦٥٠٠ where w=٠.٩
Third	٥٧٦١٠٣٦٤٠
Proposed(١)	٥٦٥٧٧٧٣٠٠ where w=٠.٩
Proposed (٢)	٥٧٦١٠٣٦٤٠

Noting the minimum total cost in the second algorithm and the proposed algorithm (١) near to the second algorithm.

٤-١ Introduction

In this thesis we summarized many conclusion and recommendation which are as follow:-

٤-٢ Conclusion

١) From the first numerical example in chapter two from table (٢-١٨) we note that the first minimum cost is the second algorithm and the second minimum cost is the proposed algorithm (١) and the third minimum cost is the crisp algorithm.

٢) From the second numerical example in chapter two from table (٢-١٨) we note that the first minimum cost is the second algorithm and the second minimum cost is the proposed algorithm (١) and the third minimum cost is the first algorithm.

٣) In the numerical application in the chapter three from table (٣-١٣) we can show that the first minimum cost is the second algorithm and the second minimum cost is the proposed algorithm (١) which satisfied the best transportation cost from the warehouses to the governorates. But the crisp algorithms, first algorithm, third algorithm and the second proposed algorithm are given the same result because we have got the same cost transportation when we transfer the fuzzy case to crisp case.

٤) We noting that the second algorithm and the first proposed algorithm gave us the best results than the other algorithms, because it dependent one weight function, while the other algorithms be oscillate.

٥) The weight function in the ranking function has the direct correlation between the transportation cost and the weight and the best weight function is when w=٠.٧

٤-٣ Recommendation

We recommend the use of genetic algorithm with ranking function and compare the result with traditional algorithms as we believe the integration of the two methods to find the optimal solution will satisfied high speed and accuracy in the solution in this case that the problem contain a large volume of data.

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Abstract

In this thesis fuzzy transportation problem in which the cost of transportation and the amount of demand and supply are fuzzy numbers studied and that the membership function is used to find the fuzzy numbers are triangular membership function.

Many algorithms have studied and proposed two algorithms have been used depending on proposed ranking function to remove fuzziness solution of fuzzy transportation problem. A numerical example and real data are used to show the efficiency of all these algorithms which used in this thesis with respect to the Vogel's algorithm with modified distribution and sure the solution by (WinQSB).

Finally, during the application of algorithms solution we studied the problem of the transfer of real whose data was obtained from Oil Products Distribution Company. Comparison between algorithms solution has been organized by minimizing costs and the results of this problem have been achieved.

الملخص

في هذا الرسالة تم دراسة مشكلة النقل الضبابية التي تكون فيها كلفة النقل وكمية الطلب والعرض هي اعداد ضبابية وان دالة الانتماء المستخدمة لايجاد الاعداد الضبابية هي الدالة المثلثية.

في هذا الرسالة تم أستخدام العديد من الخوارزميات بالاضافة الى أقتراح خوارزميتين جديدتين تعتمدان على دالة Yager لإزالة الضبابية من مشكلة النقل.

وقد تم أستخدام الامثلة العددية والبيانات الحقيقية لأظهار مدى كفاءة جميع الخوارزميات المستخدمة في الرسالة والمقارنة بينهما،حيث تم حل مشكلة النقل الهشه (crisp) بأستخدام خوارزمية فوجل بالاضافة الى الطريقة المعدلة (Modified distribution) وتم التأكد من الحل بأستخدام برنامج (Win.QSB)

اخيرا، تم تطبيق خوارزميات الحل المستخدمة في الرسالة على مشكلة نقل حقيقيه والتي تم الحصول على بياناتها من شركة توزيع المنتجات النفطية والمقارنه بين خوارزميات الحل من خلال تقليل الكلف وتم ايجاد نتائج هذة المشكلة .