MISSION REQUIREMENTS OF IMAGING SATELLITE

A THESIS

Submitted to the Military College of Engineering As A Partial Fulfillment of the Requirement for the Degree of Master of Science in Mathematics

By
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CHAPTER ONE

GENERAL INTRODUCTION AND LITERATURE SURVEY

1.1 INTRODUCTION

The design of mission plan to launch a mini-satellite into orbit for imaging purposes depends primerely on requirements and limitations. The requirements are to image the earth surface in the visible band of spectrum from orbit by a camera on board a satellite without orbit control. Th constraints depend on camera technical limitations such as resolution, imagin rates, coverage area illumination of earth surface. The selected orbit is altitude 500Km and inclination 40°. In order to launch a satellite into such orbi a multi-stage ballistic missile is required, depending on velocity, altitude a down – range requirements. The optimization of trajectory from the launchi site into injection point can be utilized by adapting the Euler–Lagran necessary conditions using initial value problem. In this process the missil considered as a point mass moving on the optimized ballistic path. verification of the path function depends on the dynamic equations of mo under the effects of aerodynamic, gravitational and thrust forces. An significant problem is how to optimize the rocket for such mission.

1.2 MISSION REQUIREMENTS

The main parameters of the spacecraft mission are related to the payload requirements and limitations. The requirement for imaging mission is to define the imaging payload specifications in terms of orbit optimal altitude and inclination angle. The altitude region which the satellite should be located while in orbit can be determined. The lower limit of designing orbit altitude is dependent on the satell lifetime as a function of altitude during the mission while the up limit depends on the weight that the booster can place into orbit the resolution of the sensor system, the trajectory analysis of launch is represented very important requirement, moreover, ground coverage area and arc length distance, while, the limitat are given by[2].

1. Diffraction effects.
2. The spherical aberration of the telescope parts (lenses and mirror
3. Transmission and refraction of solar radiation through atmosphere.
4. Limited size and stability of the platform.

1.3 TYPES OF LAUNCHERS

Strategic ballistic missiles can be divided into three general ca according to their basing modes: those that are launched from land a launched at sea and the air launch. They also can be divided accordin range into short range ballistic missiles (SRBMs), intermediate – range missiles (IRBMs) and intercontinental ballistic missile (ICBMs). are almost all of (ICBMs) range[3]. Fo landbased strategic missiles are almost all of (ICBMs) range launchers can be divided into two kinds with respect to carrying, the deals with the large launcher for puts a satellite into transfer orbit, series (1,2,3,4 and 5) and H.II[3]. While the second types deals wit launcher technology for small space – crafts, for example Minima[4].

The type of trajectory can be chosen for any vehicle according to the mission for which it's to be used, and each vehicle must be designed always with the aim of the mission in view. Trajectories are selected to achieve maximum ground range, altitude or velocity, on occasions, a combination of two or more of these parameters, for example, the long range ballistic missile require a trajectory which ensures adequate ground range. Three types of trajectory are normally used : -

1. vertical ascent

2. Inclined trajectory , also can be divided into two steps as follows :-
   i) constant turn – over rate
   ii) constant inclination to horizontal

3. Gravity turn – trajectory

Two types of trajectories are illustrated in Fig (1-1a, 1b) . Fig (1-1a) show the trajectory of three-stage ballistic missile ground launch . Fig (1-1b) is typical trajectory by air launch .[4]

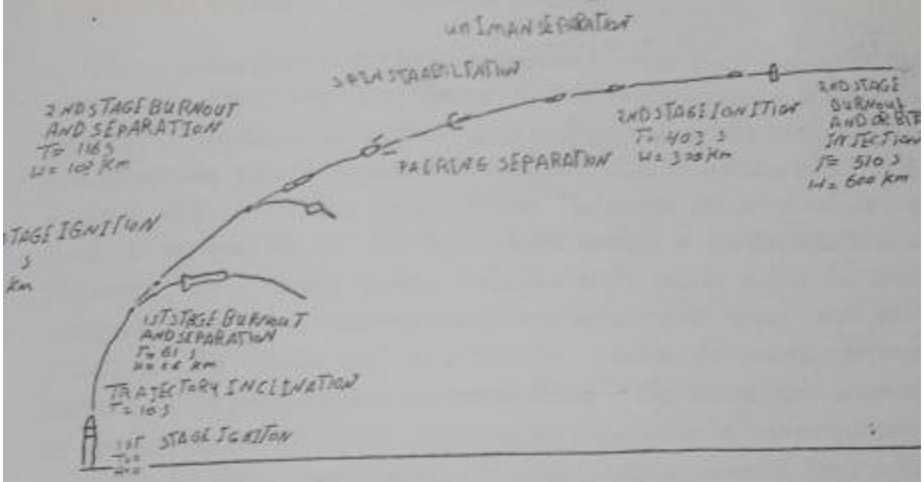

Fig (1.1a)
The mission plan of launcher from land

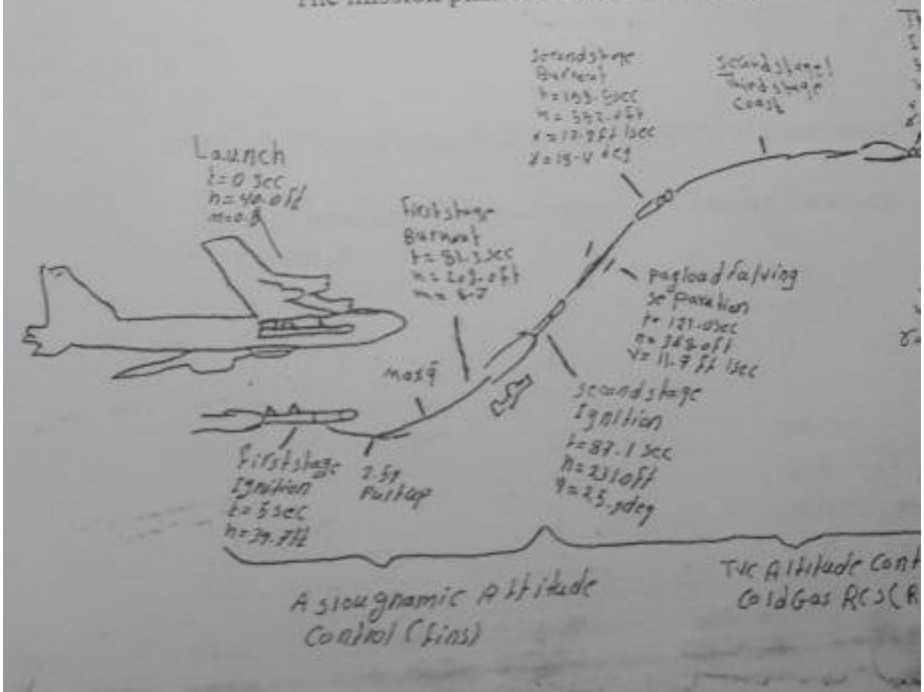

Fig (1:1b)
The mission plan of air launch

1.4 Free Flight (Coasting Phase)

The coasting phase is that phase of trajectory, which is not powered [? this case is called free flight trajectory [6]. It's usually employed in sol problem of getting the space vehicle to it's orbital altitude economically this phase the vehicle exchanges velocity for altitude so that orbital a reached with an orbital velocity deficit. The power unit of the vehic fired to increase the velocity to the level needed to put the vehicl However, the coasting phase would never be begun within the accepted limits of the atmosphere because aerodynamic forces woul disturb it . In vehicle such as a satellite launcher, the coasting b burnout of the penultimate stage and the firing of the final, apogee, st of several minutes duration . The coasting phase duration between s optimize in order to approach the required launching velocity. Typi is the coasting phase of the following launchers [3] [7]. "See Table (

Table 1.1 Typical example for coasting phase of some la

Name of launcher	No. of stage	Coasting durati
Arian .4	1	0.0
	2	1.0
	3	10.0
	4	0.0
Pegasus	1	3.5
	2	306.0
	3	0.0
H.II	1	0.0
	2	1.0
	3	10.0

1.5 CALCULUS OF VARIATIONS

Functions can be defined as correspondence which assigns a definite (real) number to each function (or curve) belonging to some class ; for example, consider all possible paths joining two given points A and B in the plane. Suppose that a particle can be moved a long any of these paths, and let the particle have a definite velocity v (x, y) at the point (x, y). Then we obtain functional by associating with each path the time particle takes to traverse the path. The branch of mathematics concerned with finding the maxima and minima of functional is called calculus of variations, more over, it would to call this subject the calculus of variations in the narrow sense , because the significance of confined to its applications to the problem of determining the extreme of functional [9] . Also, it deals with best curve fitting, the short pat between two points and trajectory optimization problem. In case of trajector optimization, the Euler–Lagrange necessary conditions play a significant role.

1.6 TRAJECTORY AND VEHICLE OPTIMIZATION

The path function can be defined as a function that gives optimum solut (minimum or maximum) to the trajectory. It does not replace the equation motion in any sense. On the contrary, its purpose is to permit integration of t equations along specific trajectory. Let us take the integrand G, involvin two variables v and γ as functions of time, whose integral J is to be maxim or minimized between two points 1 and 2.[9]

$$J=\int_{1}^{2} G(v,\dot{v},\gamma,\dot{\gamma},t)\,dt$$

The Euler necessary conditions are given by

$$\frac{d}{dt}\left(\frac{\partial G}{\partial \dot{v}}\right)-\frac{\partial G}{\partial v}=0,\text{-----------}(1.6.1a)$$

and

$$\frac{d}{dt}\left(\frac{\partial G}{\partial \dot{\gamma}}\right)-\frac{\partial G}{\partial \gamma}=0,\text{-----------}(1.6.1b)$$

In general the system described must observe some physical limitations which restrict the class of optimum solutions by constraining the possible values of G, and in this case the solution of Euler relations given above will not provide the desired answers since these will not satisfy the restraint conditions. The types of restraints which one can impose are quite varied, for example, if we wanted to consider only that class of solution providing a given change in v from 1 to 2, we would state that $\int_1^2 v\,dt = constant$. This is a perimeter restraint, which does not involve a coupling between the two dependent variables, therefore $J = \int_1^2 (G + \lambda v)dt$ where, λ is a constant and is called Lagrange – multiplier. In this case the Euler – Lagrange necessary conditions become

$$\frac{d}{dt}\left(\frac{\partial(G+\lambda v)}{\partial \dot{v}}\right) - \frac{\partial(G+\lambda v)}{\partial v} = 0, \text{------------}(1.6.2a)$$

and

$$\frac{d}{dt}\left(\frac{\partial(G+\lambda v)}{\partial \dot{\gamma}}\right) - \frac{\partial(G+\lambda v)}{\partial \gamma} = 0, \text{------------}(1.6.2b)$$

Now, the restraints can be divided into three types as follows :-

1. ***Holonomic restraint*** :-

The restraint is called holonomic if the system were restrained by a relation o the type $v = \mathbf{constant}$, then both v and $\dot{v}$ would vanish from the formulati and the system would be free only in γ. If we are working with a dynam system, we would have reduced our degrees of freedom by one.

2. ***Anholonomic restraint*** :-

If now we have a restraint such that $\dot{v} = q(t)\dot{\gamma}$ and q(t) is unknown, we not integrate to eliminate v and γ, this restraint can be introduced by defin function F as $F = \dot{v} - q(t)\dot{\gamma}$ and thus finding the extremums of

$$J = \int_1^2 (G + \lambda F)dt.$$

A restraint of this type is called anholonomic.

3. *Pseudoholonomic restraint:-*

If we now consider a restraint of the type

$$\dot{r} = p(t) + q(t)(\dot{\gamma})^b$$

Where, p(t) is some unknown function of time, and b is any number having a value of 2 or more, we see that two or more values of $\dot{r}$ can exist, satisfying the foregoing equation. Such a restraint is called pseudoholonomic restraint.

1.7 NUMERICAL METHODS OF INTEGRATION

The differential equations in general may be classified into two categories, the initial value and the boundary value problems.

In the initial value problems, the first initial conditions are required, then step – by-step solution is proceeded. The methods of solution of initial value problems can be classified into single step and multi-step methods. In the single step methods only the first initial condition is required. Such as Euler, Taylor series, Runge – Kutta, etc. There are various versions of Runge-Kutta method second order, third order and the fourth order. The fourth order Runge -Kutta method has different versions, such as Runge - Kutta – Merson of fifth order, Runge-Kutta-Felberg of six orders and Kutta-Gill method.

For more precise solutions of differential equations, the multi – step methods may be employed. Such as Adams – Moulton, Adams – Bashforth, Predicator corrector methods, Gauss–Jackson. These methods need at least four initial conditions.

In this work, the following algorithms were used for solutions of system of differential equations of motion.

1. The fourth order Runge-Kutta method:

This is a single step method for the first order differential equation of motion is as follows: [10]

$$\frac{dy}{dx} = f(x_j, y_j)$$

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The difference formula is

$$y_{i+1} = y_i + \frac{k_1 + 2(k_2 + k_3) + k_4}{6} \quad \text{----------(1.7.1)}$$

Where k1, k2, k3, k4 are fourth order Runge Kutta parameters given by :

$$k_1 = \delta \times f(x_i, y_i) \quad \text{--------------(1.7.2a)}$$
$$k_2 = \delta \times f(x_i + \delta/2, y_i + k_1/2) \quad \text{-------(1.7.2b)}$$
$$k_3 = \delta \times f(x_i + \delta/2, y_i + k_2/2) \quad \text{-------(1.7.2c)}$$
$$k_4 = \delta \times f(x_i + \delta, y_i + k_3) \quad \text{-----------(1.7.2d)}$$

where δ is the step size and f (x_0 , y_0) is initial solution

3. Kutta- Gill method

This is a single step method. It needs only first initial condition for each variable.

The numerical integration procedure is expressed as follows [11]

$$y_{1,1} = y_{1,0} + \frac{1}{2}(k_{1,1} - 2q_{1,0}) \quad \text{-----------(1.7.3a)}$$
$$y_{1,2} = y_{1,1} + (1 - \frac{1}{\sqrt{2}})(k_{1,2} - q_{1,1}) \quad \text{-----------(1.7.3b)}$$
$$y_{1,3} = y_{1,2} + (1 + \frac{1}{\sqrt{2}})(k_{1,3} - 2q_{1,2}) \quad \text{-------(1.7.3c)}$$
$$y_{1,4} = y_{1,3} + \frac{1}{6}(k_{1,4} - 2q_{1,3}) \quad \text{-----------(1.7.3d)}$$

The k parameters are given by:

$$k_{1,1} = \delta \times f(y_{1,0}, P_{1,0}) \quad \text{------------(1.7.4a)}$$
$$k_{1,2} = \delta \times f(y_{1,1}, P_{1,1}) \quad \text{------------(1.7.4b)}$$
$$k_{1,3} = \delta \times f(y_{1,2}, P_{1,2}) \quad \text{------------(1.7.4c)}$$
$$k_{1,4} = \delta \times f(y_{1,3}, P_{1,3}) \quad \text{------------(1.7.4d)}$$

Where $P_{i,0}=P_i(t_0)$ -- (1.7.5a)

$P_{i,1}=P_{i,2}=P_i(t_0+\delta/2)$ ---------------------------------- (1.7.5b)

$P_{i,3}=P_i(t_0+\delta)$ -- (1.7.5c)

And the q parameters are given by :

$$q_{i,1}=q_0+3\left\{\frac{1}{2}(k_{1,i}-2q_0)\right\}-\frac{1}{2}k_1 \quad \text{------------ (1.7.6a)}$$

$$q_{i,2}=q_1+3\left\{(1-\frac{1}{\sqrt{2}})(k_{2,i}-q_1)\right\}-(1-\frac{1}{\sqrt{2}})k_2 \quad \text{-------- (1.7.6b)}$$

$$q_{i,3}=q_2+3\left\{(1+\frac{1}{\sqrt{2}})(k_{3,i}-q_2)\right\}-(1+\frac{1}{\sqrt{2}})k_3 \quad \text{-------- (1.7.6c)}$$

$$q_{i,4}=q_3+3\left\{\frac{1}{6}(k_{4,i}-2q_3)\right\}-\frac{1}{2}k_4 \quad \text{------- (1.7.6d)}$$

Where $q_{i,4}(t_0)=q_{i,0}(t_0+\delta)$

3. Adams-Moulton Predictor – corrector method.

This is a multi-step method. It needs at least four initial conditi… numerical integration procedure can be divided in to two steps as follow…

1. Predictor

$$y_{j+1}=y_j+\frac{\delta}{24}\Big[55p(x_j,y_j,P_j)-59p(x_{j-1},y_{j-1},P_{j-1})+37p(x_{j-2},y_{j-2}\ldots$$

$$-9p(x_{j-3},y_{j-3},P_{j-3})\Big] \text{--}$$

$$p_{j+1}=p_j+\frac{\delta}{24}\Big[55F(x_j,y_j,P_j)-59F(x_{j-1},y_{j-1},P_{j-1})+37F(x_{j-2},y_{j-2}\ldots$$

$$-9F(x_{j-3},y_{-3},P_{j-3})\Big] \text{-}$$

NASA [illegible] introduced a program in (6DOF) to optimize trajectory

An experiment rig of servo design and control and it's correlation with dynamics of short range missiles was introduced in [19]

The simulation of various launchers and launching site was introduced in [3]. The H-II launch vehicle which has the capability to launch a two ton class satellite into the geostationary orbit was introduced in [7]. Chassan [24], introduced the simulation of rigid body motion in space and the simulation of the flight vehicle motion.

One of the essential mission requirements is launching a satellite into the required orbit. An optimized trajectory of the launchers is required from initial point to the final point. Therefore, the application of variational calculus to optimum trajectories may be classified into two parts. The initial value problems which are introduced in [9] and [18], and boundary value problems are introduced in [8] and [21]. The optimum methods are introduced in [22] and [23].

Shruster and et.al [24] discussed a method of optimizing the angle of attack to achieve maximum range for un powered gliding flight. Trajectory optimization for two stages winged flight system for space transportation was introduced by [25]. This flight system used the atmosphere to produce lift and thrust. It consisted of two stages, the currier stage, which was equipped with air breathing turbo, ramjet engines and the orbital stage using rocket engines.

Vehicle structure-propellant optimization for multi – stage missile used to launch a satellite into the earth orbit was introduced by [5] and [27]. The numerical integration methods are introduced in [11], [29] and [12]. The Re-entry vehicle dynamics introduced in [31], [32] and [33]. At the end of the 20^{th} century small launchers technology for small space craft, introduced in the ref [4].

CHAPTER FOUR

COMPUTER SIMULATION AND DISCUSSION

4.1 INTRODUCTION

In order to evaluate the results clearly , it is suitable to handle the result analysis in three main parts . The first part will describe the dynamics of rigid body (projectile) , while the second part deals with the trajectory optimization analysis and the third part deals with atmospheric models .

4.2 DYNAMICS OF RIGID BODY (PROJECTILE)

This section divided into four phases as follows:

PHASE 1 2DOF

In this phase, computer simulation had been developed to solve the system of Eqs (2.3.3 – 2.3.6) simultaneously by using Runge-kutta forth order method.

The initial conditions which have been used in this phase are:-

1 Trajectory angle θ =85°,70°,45°

2 Velocity V_o=500 m/sec

3 Altitude y = 0 km

4 Down-range x =0 km

The simulation results are shown in Figs (4.1-4.6)

Now, the main motor thrust is given by:

$T_{main} = T - (P - P_{at})EA(i)$

where:

P is the chamber pressure

P_{at} is the atmosphere pressure

EA(i) is the cross – sectional exhaust area of engine of each stage.

$C_{(i,j)}$ The mass flow coefficients of the main engine.

2. The Booster thrust

In most multi-stage missile a number of boosters are attached to the main engine in order to increase the lift-off force.

The thrust vector of each booster is given by:

$T_{sB} = I_{sB}(i)\dot{m}_B$

where $I_{sB}(i)$ is the specific impulse of each booster in stage i

$\dot{m}_B$ is the mass flow rate

[illegible] if the mass flow coefficients of each booster in each stage are available then the booster's thrust is given by the following polynomial:

$T_{sB} = C_{B(1,i)} + [illegible] C_{B(2,i)} + [illegible] C_{B(3,i)} + [illegible] C_{B(4,i)} + [illegible] C_{B(5,i)}$

where $C_{B(1,i)}$ ... the mass flow coefficients of each booster

Now the total thrust of each booster is given by:

$T_{Boost} = (T_{sB} - (P - P_{at})EA_B(i))N_B$

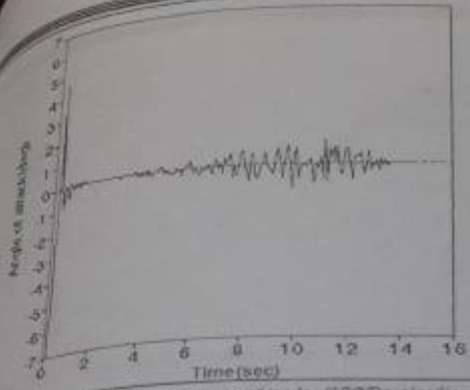
where

$EA_B(i)$ is the cross –sectional exhaust area of each booster in each stage.

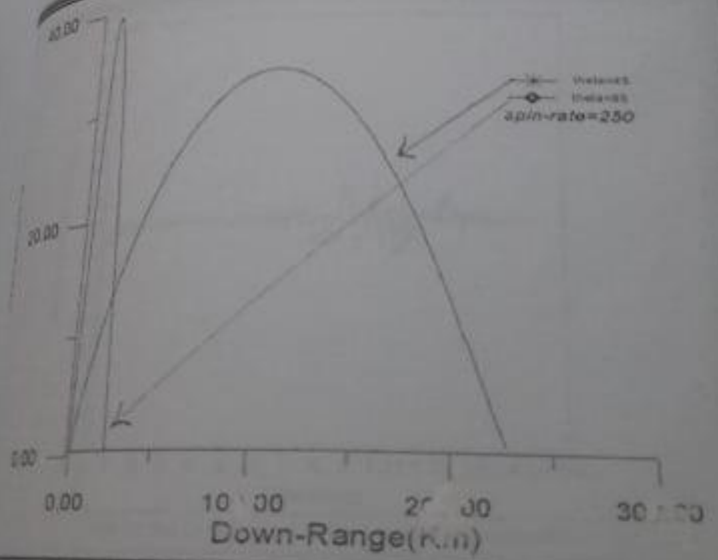
N_B is the number of boosters in each stage.

The sum of main motor thrust and those of the boosters of each stage is given by:

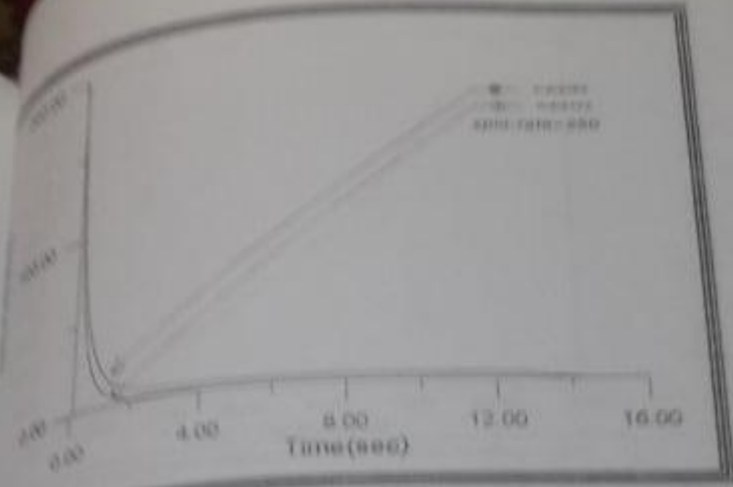
$T = T_{main} + T_{Boost}$



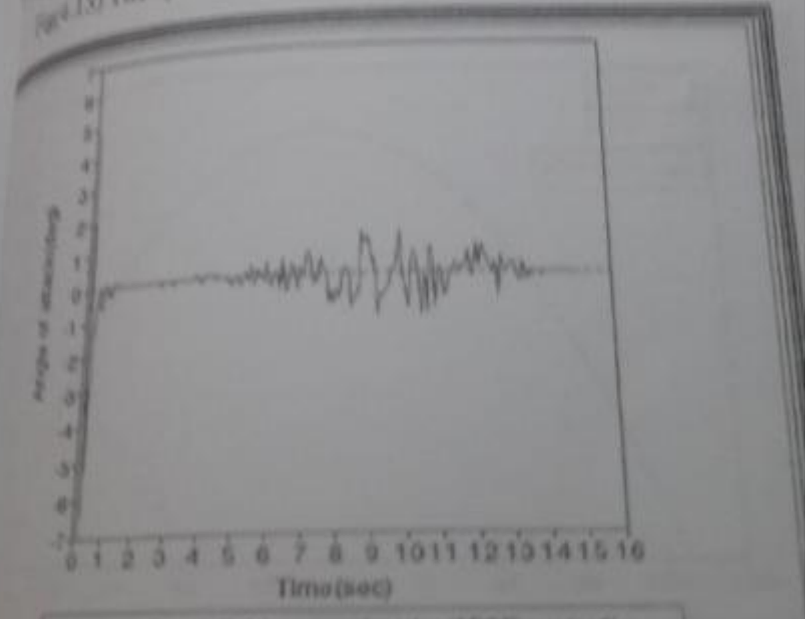
Fig(4.11) The (A.O.A.) against time for (6DOF)projectile



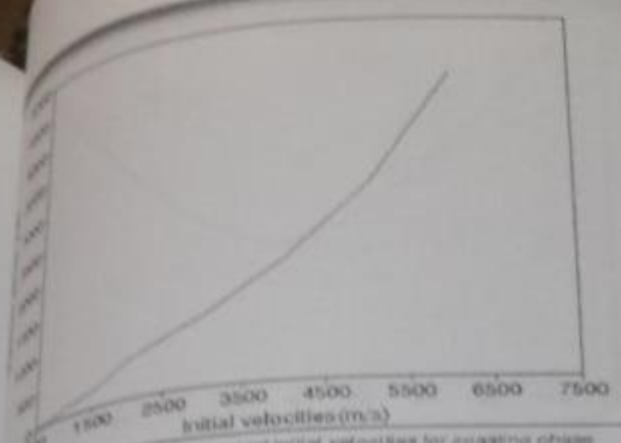
Fig(4.12) The altitude against down-range for (6DOF) Projectile



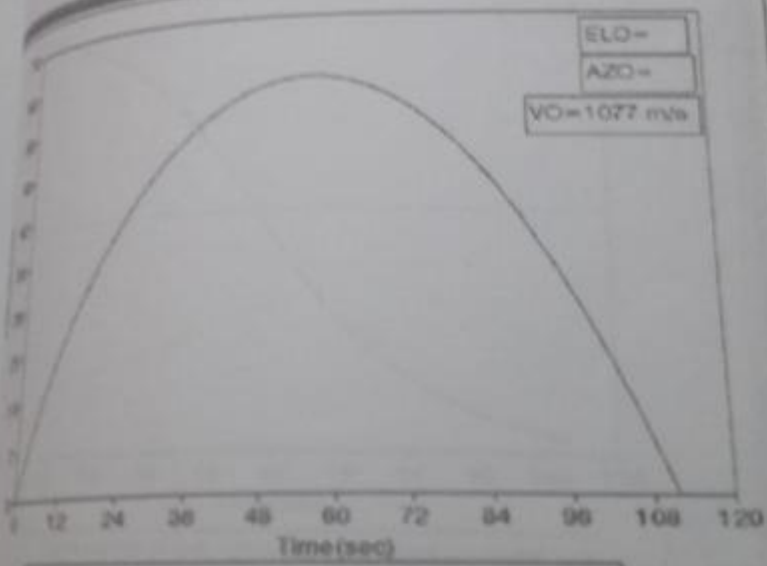
(Fig 4.13) The spin-rate against time for (6DOF) projectile



fig(4.14) The (A.O.A.) against time for (6DOF) projectile



[illegible] The max altitude against initial velocities for coasting phase



[illegible] The altitude for coasting phase against time

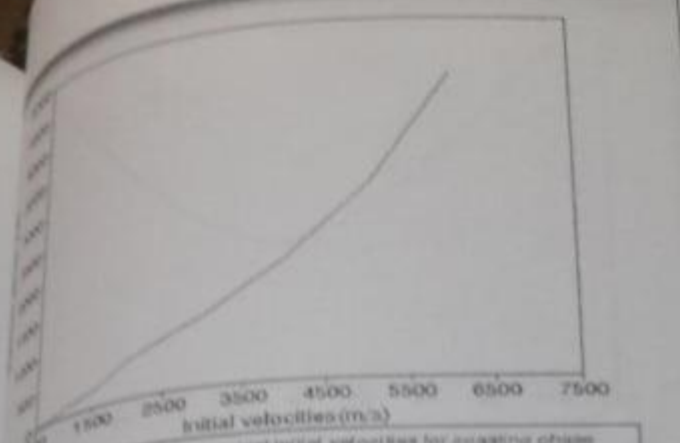
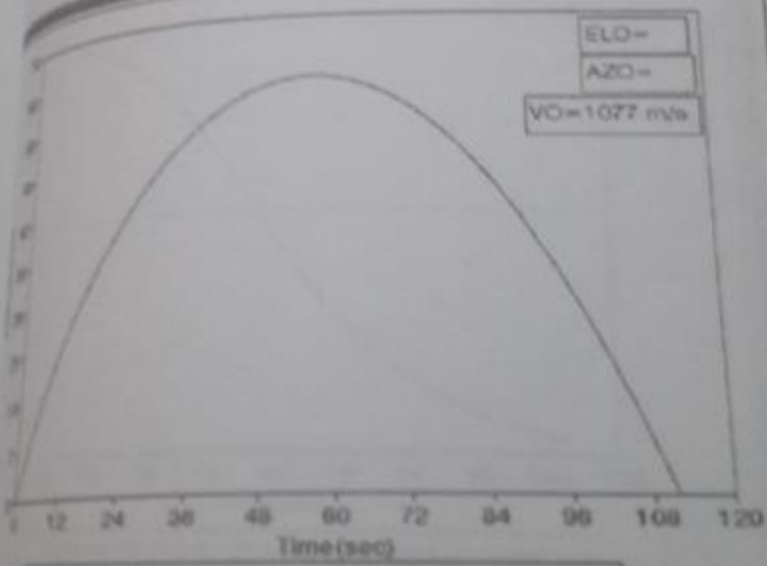
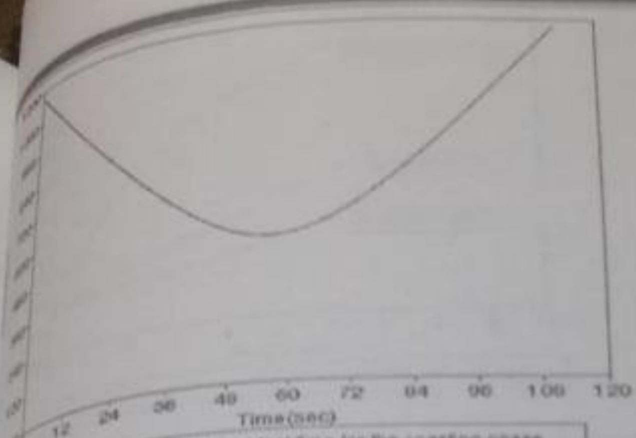


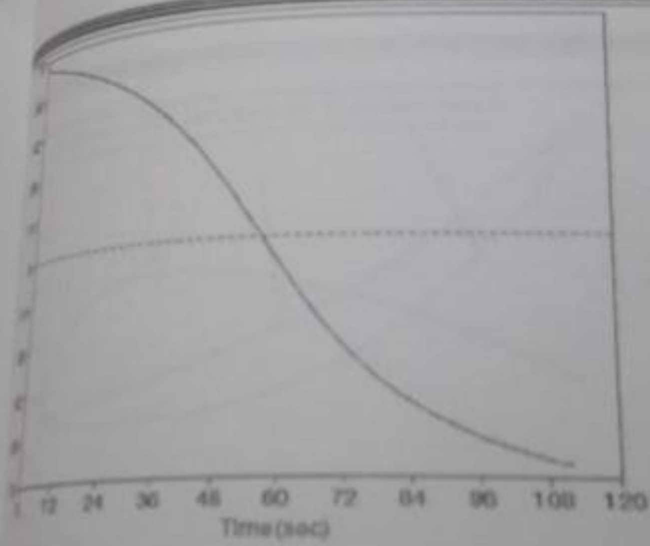
Figure [illegible] The max altitude against initial velocities for coasting phase



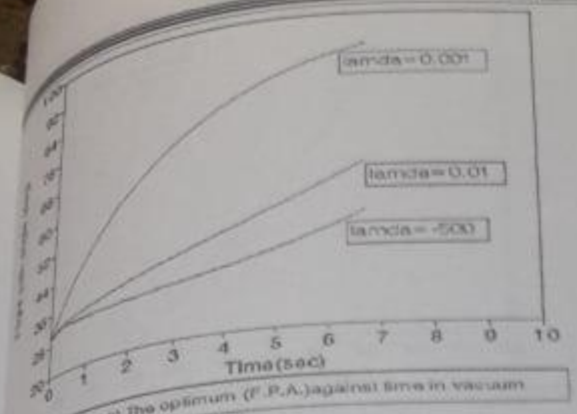
Fig[illegible] The altitude for coasting phase against time



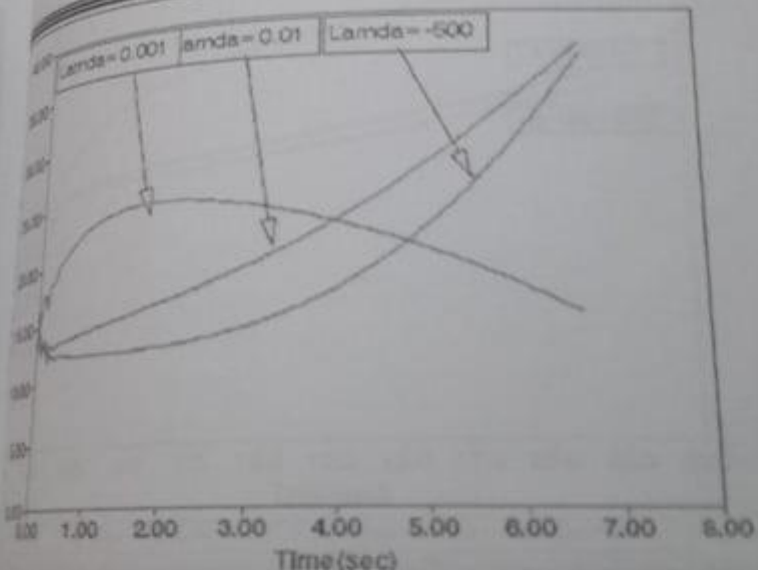
Fig(4.17) - The velocity against time for the coasting phase



Fig(4.18) The (F.P.A) against time for coasting phase

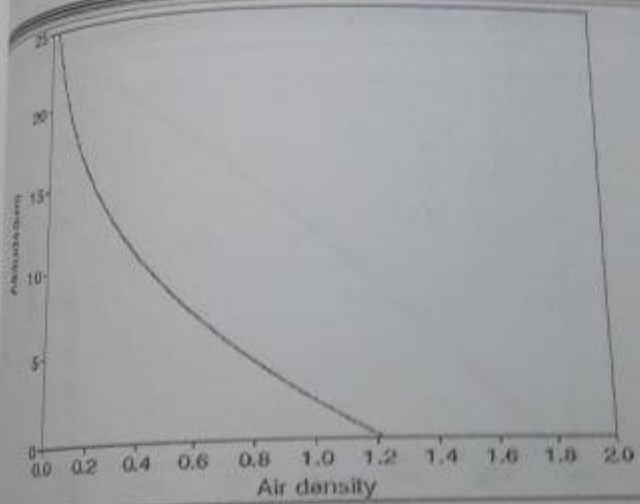


Fig(4.?) The optimum (F.P.A.)against time in vacuum

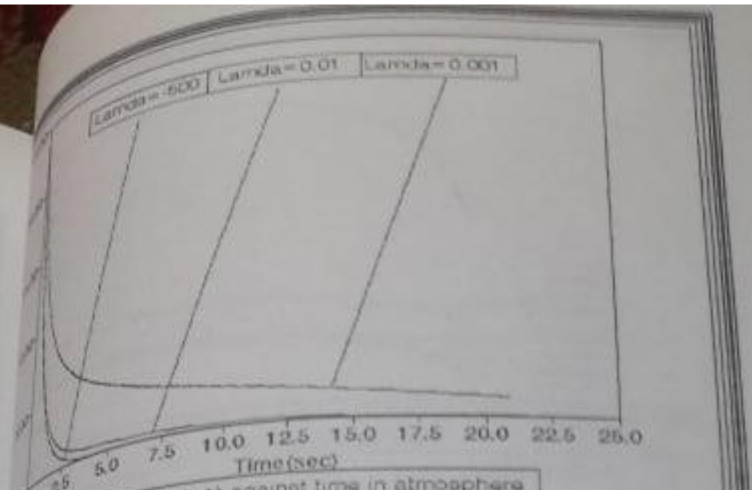


Fig(4.?) The optimum (A.O.A)against time in vacuum

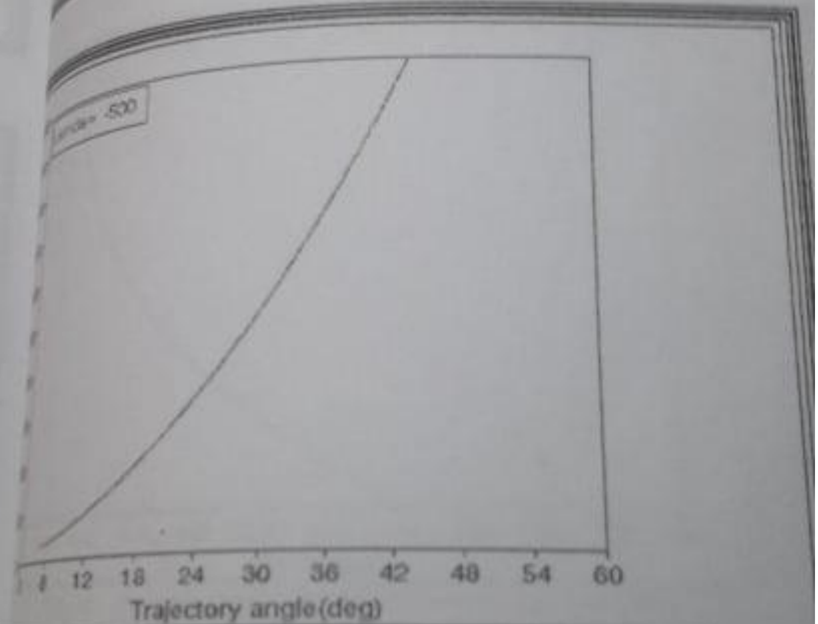
Fig(4.15)The optimized velocity requirement to launch payload(235 Kg) into circler orbit against lagrange multiplier



Fig(4.16):- Air density against altitude for short range projectile

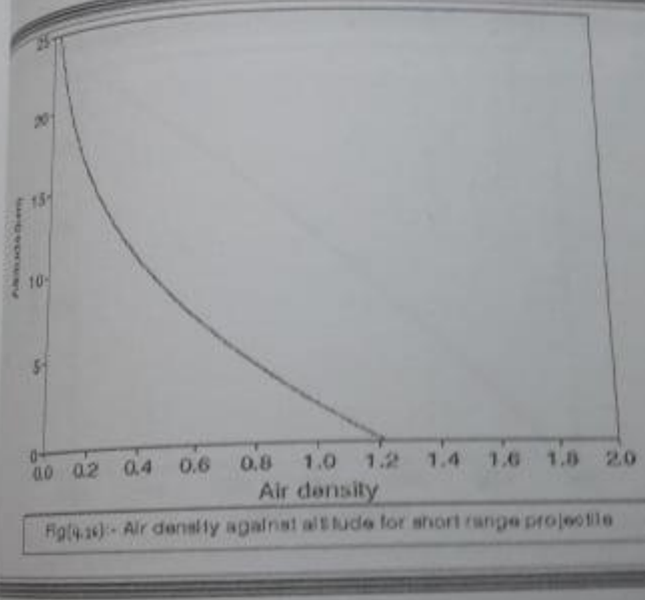


Fig(4-2) The (A.C.A) against time in atmosphere

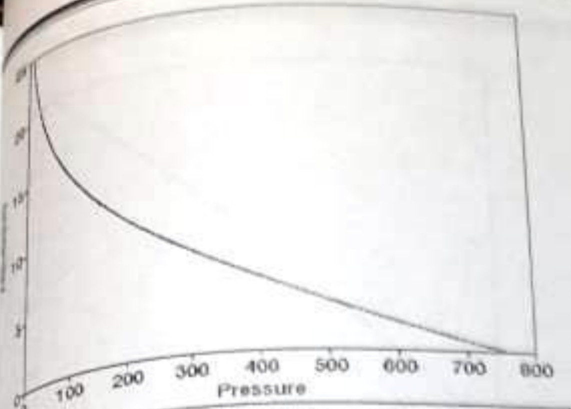


...the optimum altitude against trajectory angle in atmosphere

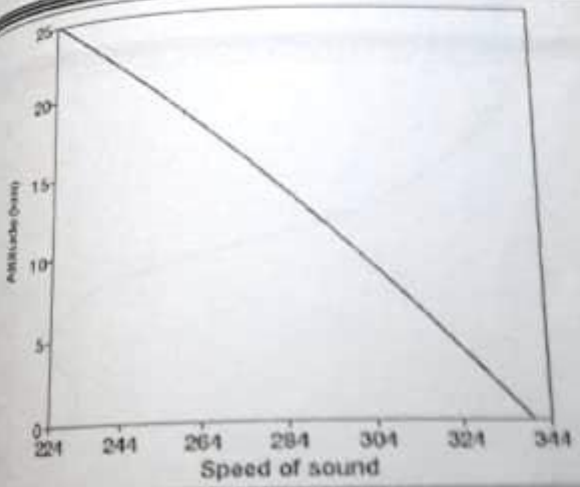
Fig(4.15)The optimized velocity requirement to launch payload(235 Kg) into circler orbit against lagrange multiplier



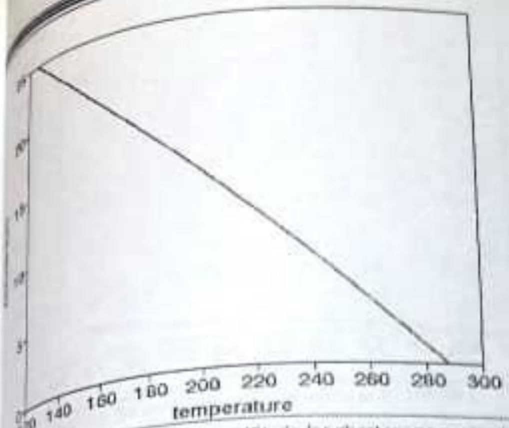
Fig(4.16):- Air density against altitude for short range projectile



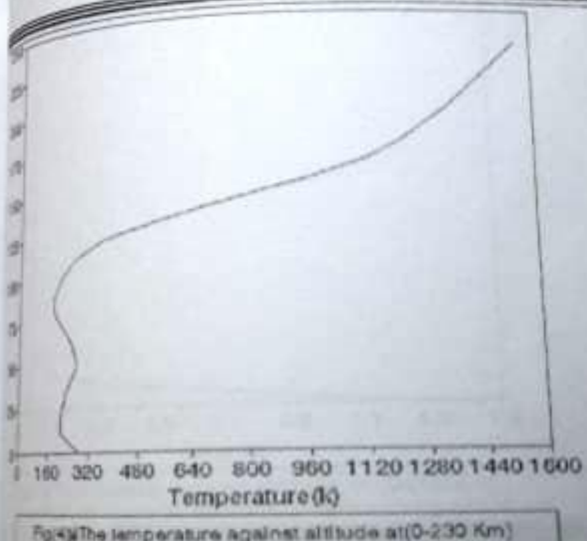
Fig(5.21):- the pressure against altitude for short range projectile



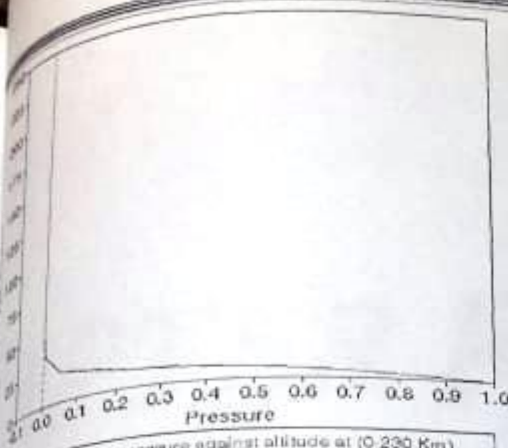
Fig(5.22):-Speed of sound against altitude for short range projectile



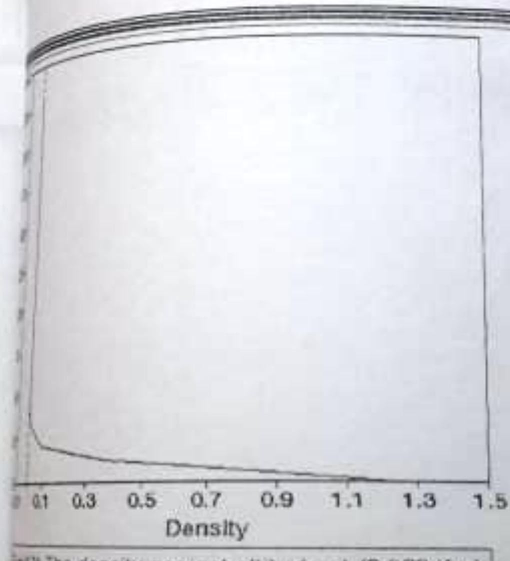
[illegible]:- The temperature against altitude for short range projectile



Fig(4)The temperature against altitude at(0-230 Km)



Fig(4.3) The pressure against altitude at (0-230 Km)



Fig(4.3) The density against altitude at (0-230 Km)

CHAPTER FIVE

...CLUSION AND FUTURE WORK

...CLUSION

...llowing concluding remarks may be drawn from the present ...k:

...l path of ballistic missiles is considered as a point mass traversing ...ic path of the projectile, and that the missile should be follow this ...

...ind has significant effect on the motion of the projectile, such that ...ase or decrease according to the wind direction.

...f spinning projectiles, the down range increases slightly as the spin ...es, and the path would be more stable.

...n of vehicle parameters (structure, propellent, payload and lift-off ...n be attained when the sum of velocities at the proposed stages equal ...ired orbital velocity at a given altitude.

...matical method of trajectory optimization (Euler-Lagrange ...conditions) shows a perfect agreement with the modern optimal ...and control system of ballistic missiles.

...al trajectory of long-range missiles can be obtained by solving ...ously the system of equations dynamic, kinematics and the path ... of the trajectory.

...ic effects of ballistic missiles are very small in compartion to the ...meters (10%).

...ing missiles are launched from height altitude (more than 15km) ...itial elevation angle (~ 30°) in order to reduce aerodynamic and ...ects of the missile body.

...launching velocity will be increased due to the velocity of the ...hicle (Ramjet).

5.2 FUTURE WORK

Within the frame work of the thesis there appeared future work to be performed, which can be summarized as follows:

- Including another mission requirement for investigation such as the satellites lit time, ground coverage of sensors, illuminations of satellite and targets, etc.
- Trajectory analysis of (3DOF) and (6DOF) of multi-stage launchers with optimal control systems.
- Determination of orbit parameters at injection point.
- Aerodynamic and gravitational forces will be introduced in the vehicle optimization problem.
- Dynamic analysis of separation between stages.
- Guidance and control systems for air launch.