Derivatives and Integrations Involving Inverse Trigonometric Functions

$$\frac{d}{dx}[\sin^{-1}x] = \frac{1}{\sqrt{1-x^2}} \Rightarrow \frac{d}{dx}[\sin^{-1}u] = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$$

$$\frac{d}{dx}[\cos^{-1}x] = \frac{-1}{\sqrt{1-x^2}} \Rightarrow \frac{d}{dx}[\cos^{-1}u] = \frac{-1}{\sqrt{1-u^2}}\cdot\frac{du}{dx}$$

$$\frac{d}{dx}[\tan^{-1}x] = \frac{1}{1+x^2} \Rightarrow \frac{d}{dx}[\tan^{-1}u] = \frac{1}{1+u^2}\cdot\frac{du}{dx}$$

$$\frac{d}{dx}[\cot^{-1}x] = \frac{-1}{1+x^2} \Rightarrow \frac{d}{dx}[\cot^{-1}u] = \frac{-1}{1+u^2}\cdot\frac{du}{dx}$$

$$\frac{d}{dx}[\sec^{-1}x] = \frac{1}{x\sqrt{x^2-1}} \Rightarrow \frac{d}{dx}[\sec^{-1}u] = \frac{1}{u\sqrt{u^2-1}}\cdot\frac{du}{dx}$$

$$\frac{d}{dx}[\csc^{-1}x] = \frac{-1}{x\sqrt{x^2-1}} \Rightarrow \frac{d}{dx}[\csc^{-1}u] = \frac{-1}{u\sqrt{u^2-1}}\cdot\frac{du}{dx}$$

Ex:- Find dy/dx if $y = \sin^{-1}(x^3)$

Sol:-

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-(x^3)^2}}\cdot 3x^2 = \frac{3x^2}{\sqrt{1-x^6}}$$

Ex:- Find dy/dx if $y = \sec^{-1}(x^2 - \frac{1}{2}x)$

Sol:-

$$\frac{dy}{dx} = \frac{1}{(x^2-\frac{1}{2}x)\sqrt{(x^2-\frac{1}{2}x)^2-1}}\cdot(2x-\tfrac{1}{2})$$

$$= \frac{\frac{1}{2}(4x-1)}{(x^2-\frac{1}{2}x)\sqrt{x^4-x^3+\frac{1}{4}x^2-1}} = \frac{4x-1}{2(x^2-\frac{1}{2}x)\sqrt{x^4-x^3+\frac{1}{4}x^2-1}}$$

$$\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + C$$

$$\int \frac{dx}{1+x^2} = \tan^{-1} x + C$$

$$\int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1} x + C$$

Ex:- Evaluate $\int \frac{dx}{1+3x^2}$

Sol:-

let $u = \sqrt{3}\,x \Rightarrow du = \sqrt{3}\,dx \Rightarrow \frac{1}{\sqrt{3}} du = dx$

$$\therefore \int \frac{dx}{1+3x^2} = \frac{1}{\sqrt{3}} \int \frac{du}{1+u^2} = \frac{1}{\sqrt{3}} \tan^{-1} u + c = \frac{1}{\sqrt{3}} \tan^{-1}(\sqrt{3}x) + C$$

Ex: Evaluate $\int \frac{dx}{a^2+x^2}$; $a \neq 0$; a Constant.

Let $x = au \Rightarrow dx = a\,du$.

$$\therefore \int \frac{dx}{a^2+x^2} = \int \frac{a\,du}{a^2+a^2u^2} = \int \frac{a\,du}{a^2(1+u^2)} = \frac{1}{a} \int \frac{du}{1+u^2}$$

$$= \frac{1}{a} \tan^{-1} u + C = \frac{1}{a} \tan^{-1} \frac{x}{a} + C.$$

From above Ex. we get:-

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1} \frac{x}{a} + C$$

$$\int \frac{dx}{a^2+x^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$$

$$\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a} \sec^{-1} \frac{x}{a} + C \quad ;(a>0)$$

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Exercises:-

In Exercises ① - ⑱ Find dy/dx.

① $y=\sin^{-1}(\frac{1}{3}x)$ ② $y=\cos^{-1}(2x+1)$ ③ $y=\tan^{-1}(x^2)$

④ $y=\cot^{-1}(\sqrt{x})$ ⑤ $y=\sec^{-1}(x^7)$ ⑥ $y=\csc^{-1}(4x)$

⑦ $y=(\tan x)^{-1}$ ⑧ $y=\dfrac{1}{\tan^{-1}x}$ ⑨ $y=\sin^{-1}(1/x)$

⑩ $y=\cos^{-1}(\cos x)$ ⑪ $y=\sqrt{\cot^{-1}x}$ ⑫ $y=x^2(\sin^{-1}x)^3$

⑬ $y=\sin^{-1}x+\cos^{-1}x$ ⑭ $y=\sec^{-1}x+\csc^{-1}x$ ⑮ $y=\tan^{-1}(\frac{1-x}{1+x})$

⑯ $y=(1+x\csc^{-1}x)^{10}$ ⑰ $y=\tan^{-1}\sqrt{\frac{1-x}{1+x}}$ ⑱ $y=\sin^{-1}(x^2\cos^{-1}x)$

sol. of ⑧ $y=\dfrac{1}{(\tan^{-1}x)}$

$$y=(\tan^{-1}x)^{-1} = -(\tan^{-1}x)^{-2} * \frac{1}{1+x^2}$$

$$= \frac{-1}{(1+x^2)(\tan^{-1}x)^2}.$$

Sol. of ⑱ $y=\sin^{-1}(x^2\cos^{-1}x)$

$$\frac{dy}{dx}=\frac{1}{\sqrt{1-(x^2\cos^{-1}x)^2}}\cdot\left[x^2\cdot\frac{-1}{\sqrt{1-x^2}}+2x\cos^{-1}x\right]$$

$$=\frac{\frac{-x^2}{\sqrt{1-x^2}}+2x\cos^{-1}x}{\sqrt{1-x^4(\cos^{-1}x)^2}}.$$

⑲ Find $\frac{dy}{dx}$ by implicit differentiation for:-

ⓐ $x^3+x\tan^{-1}y=\sec^{-1}y$ ⓑ $\sin^{-1}(xy)=\cos^{-1}(x-y)$

sol. of ⓑ

$$\frac{1}{\sqrt{1-x^2y^2}}\cdot\left(x\frac{dy}{dx}-y\right)=\frac{-1}{\sqrt{1-(x-y)^2}}\cdot\left(1-\frac{dy}{dx}\right)$$

$$\Rightarrow \frac{x\frac{dy}{dx}-y}{\sqrt{1-x^2y^2}}=\frac{\frac{dy}{dx}-1}{\sqrt{1-x^2+2xy-y^2}}$$

In Exercises 20 – 36 evaluate the integrals:

20. $\int_0^{1/\sqrt{2}} \frac{dx}{\sqrt{1-x^2}}$

21. $\int_{-1}^{1} \frac{dx}{1+x^2}$

22. $\int \frac{dx}{1+16x^2}$

23. $\int_{\sqrt{2}}^{\sqrt{2}} \frac{dx}{x\sqrt{x^2-1}}$

24. $\int_{-\sqrt{2}}^{-2/\sqrt{3}} \frac{dx}{x\sqrt{x^2-1}}$

25. $\int \frac{dx}{\sqrt{1-4x^2}}$

26. $\int \frac{dx}{x\sqrt{9x^2-1}}$

27. $\int_1^3 \frac{dx}{\sqrt{x}(x+1)}$

28. $\int \frac{t}{t^4+1}\, dt$

29. $\int \frac{\sec^2 x\, dx}{\sqrt{1-\tan^2 x}}$

30. $\int \frac{\sin\theta\, d\theta}{\cos^2\theta+1}$

31. $\int \frac{dx}{x\sqrt{1-4x^2}}$

32. $\int \frac{dx}{\sqrt{9-x^2}}$

33. $\int \frac{dx}{5+x^2}$

34. $\int \frac{dx}{x\sqrt{x^2-\pi}}$

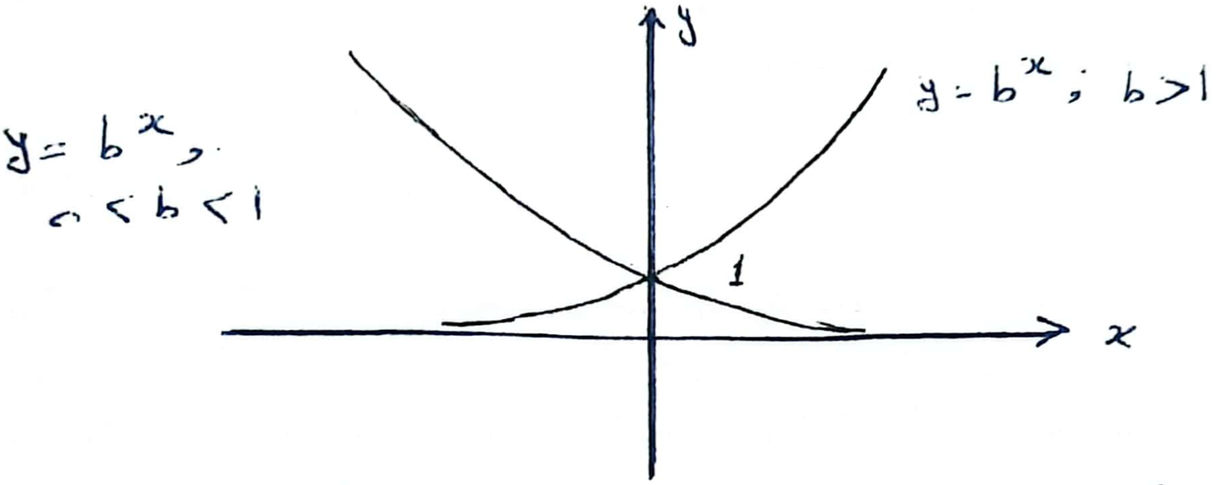
35. $\int \frac{dx}{\sqrt{9-4x}}$

36. $\int \frac{dy}{y\sqrt{5y^2-3}}$.

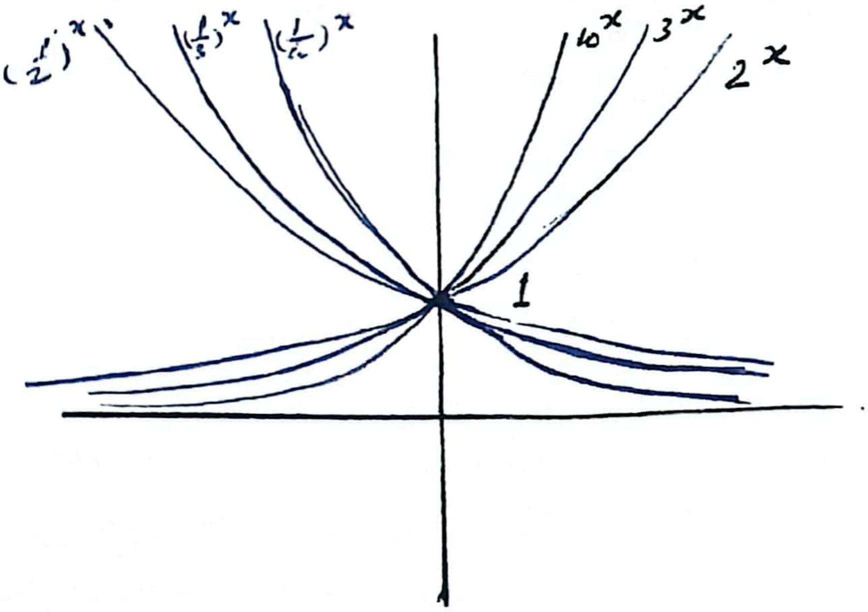
Sol. of 24

Exponential and Logarithmic Functions:

A function of the form $f(x) = b^x$; where $b > 0$ and $b \neq 1$ is called an exponential function with the base b.



we can plot some of the functions in exponential forme as:-



and If $b = 1$ then the function b^x is a constant function since $b^x = 1^x = 1$ for any x and the graph of b^x is a horizontal line pass through the point $(0, 1)$.

Theorem :- If $b > 0$ and $b \neq 1$; then :-

① The function $f(x) = b^x$ is defined for all real values of x; so its natural domain is $(-\infty, +\infty)$

② The function $f(x) = b^x$ is continuous on the interval $(-\infty, +\infty)$ and its range is $(0, +\infty)$

Logarithms :-

If $b > 0$ and $b \neq 1$, then for positive values of the Logarithm to the base b of x is defined by :- $\log_b x$

and is defined to be that exponent to which b must be raised to produce x.

For example

$\log_{10} 100 = 2$ since $10^2 = 100$

$\log_{10} (1/1000) = -3$ since $10^{-3} = \frac{1}{1000}$

$\log_2 16 = 4$ since $2^4 = 16$

$\log_b 1 = 0$ since $b^0 = 1$

$\log_b b = 1$ since $b^1 = b$

remark :- ① where the logarithms with base 10 called common logarithm and for such logarithms it is usual to suppress explicit reference to the base and write $\log x$ rather than $\log_{10} x$.

i.e

$$\log x \cong \log_{10} x.$$

② logarithm with base 2 have played a role in computer science

③ the natural logarithms which have an irrational base denoted by the letter e where $e \approx 2.718282$.

3 - 31

notice:-

$$e = \lim_{x \to +\infty} \left(1 + \frac{1}{x}\right)^x$$

and

$$e = \lim_{x \to -\infty} \left(1 + \frac{1}{x}\right)^x$$

So

$$e = \lim_{x \to 0} (1 + x)^{1/x}$$

The natural logarithm of x is denoted by $\ln x$ where

$$\log_e x = \ln x \quad \text{and} \quad y = \ln x \Rightarrow x = e^y, \; x > 0, \; y \in \mathbb{R}$$

that means

$\ln 1 = 0$, $\ln e = 1$, $\ln 1/e = -1$, $\ln (e^2) = 2$

since $e^0 = 1$; $e^1 = e$; $e^{-1} = \frac{1}{e}$; $e^2 = e^2$

now :-

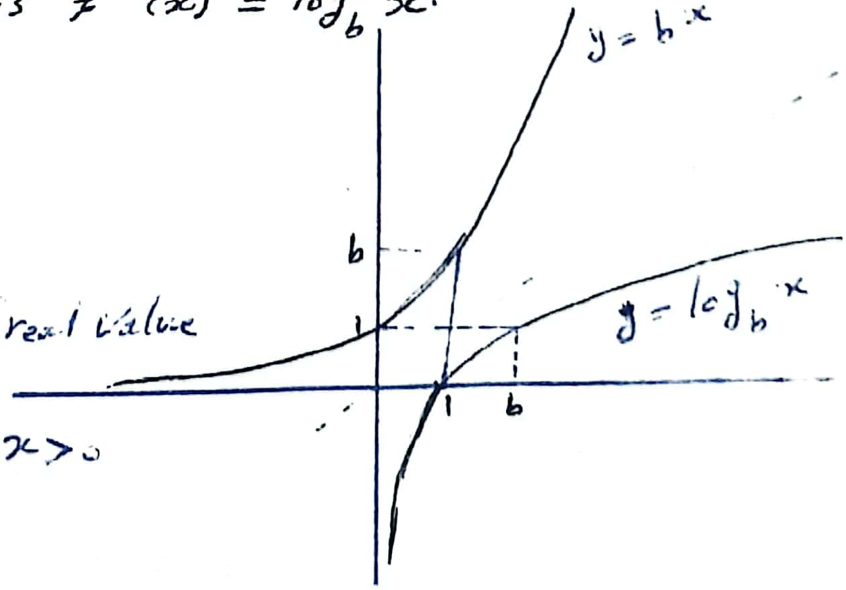
since $\log_b (b^x) = x$ for all real values of x.

and

$b^{\log_b x} = x$ for $x > 0$

Then $f(x) = b^x$ is $f^{-1}(x) = \log_b x$.

i.e



So:-

$\ln (e^x) = x$ for all real value

$e^{\ln x} = x$ for all $x > 0$

So $\log 10^x = x$, $10^{\log x} = x$

Theorem :-
If $b > 0$ and $b \neq 1$ then :-

$b^0 = 1$ $\qquad$ $\log_b 1 = 0$
$b^1 = b$ $\qquad$ $\log_b b = 1$
range $b^x = (0, +\infty)$ $\qquad$ domain $\log_b x = (0, +\infty)$
domain $b^x = (-\infty, +\infty)$ $\qquad$ range $\log_b x = (-\infty, +\infty)$
$y = b^x$ is continuous on $(-\infty, +\infty)$ $\qquad$ $y = \log_b x$ is continuous on $(0, +\infty)$

Theorem :- Algebric properties of Logarithms :-
If $b > 0$, $b \neq 1$, $a > 0$, $c > 0$ and r any real numbers then :-

$$\log_b (ac) = \log_b a + \log_b c$$

$$\log_b (a/c) = \log_b a - \log_b c$$

$$\log_b (a^r) = r \log_b a$$

$$\log_b (1/a) = -\log_b a$$

Ex :- Find x such that :-
① $\log x = \sqrt{2}$ ⓑ $\ln(x+1) = 5$ ⓒ $5^x = 7$.

① $x = 10^{\sqrt{2}} \approx 25.95$
② $x+1 = e^5 \Rightarrow x = e^5 - 1 \approx 147.41$
③ $x \ln 5 = \ln 7 \Rightarrow x = \frac{\ln 7}{\ln 5} \approx 1.21$

3-5[illegible]

Ex: Solve $\frac{e^x - e^{-x}}{2} = 1$ for x.

$$\frac{e^x - e^{-x}}{2} = 1 \Rightarrow e^x - e^{-x} = 2$$

$$\Rightarrow e^x - \frac{1}{e^x} = 2$$

$$\Rightarrow e^{2x} - 1 = 2e^x$$

$$\Rightarrow e^{2x} - 2e^x + 1 = 0$$

$$\Rightarrow (e^x)^2 - 2e^x + 1 = 0$$

$$\Rightarrow u^2 - 2u + 1 = 0 \qquad u = e^x$$

$$u = \frac{2 \mp \sqrt{4+4}}{2} = \frac{2 \mp \sqrt{8}}{2} = 1 \mp \sqrt{2}$$

$$\therefore e^x = 1 \mp \sqrt{2} \Rightarrow e^x = 1 + \sqrt{2} \quad \text{since } e^x \geq 0$$

$$\therefore e^x = 1 + \sqrt{2} \Rightarrow \ln e^x = \ln(1+\sqrt{2})$$

$$\Rightarrow x = \ln(1+\sqrt{2}) \approx 0.881.$$

remark:-

1. $$\log_b x = \frac{\ln x}{\ln b}$$

2. $$\lim_{x \to +\infty} e^x = +\infty \quad ; \quad \lim_{x \to \infty} \ln x = +\infty$$

3. $$\lim_{x \to -\infty} e^x = 0 \quad ; \quad \lim_{x \to 0^+} \ln x = -\infty$$

4. $$\lim_{x \to +\infty} e^{-x} = 0 \quad ; \quad \lim_{x \to -\infty} e^{-x} = +\infty$$

Derivatives and Integrals Involving Logarithmic and Exponential Functions:-

$$\boxed{\begin{aligned} \frac{d}{dx}[\log_b x] &= \frac{1}{x\ln b} \quad ; x>0 \\ \frac{d}{dx}[\ln x] &= \frac{1}{x} \quad ; x>0 \end{aligned}}$$

Using the chain rule we get:-

$$\frac{d}{dx}[\log_b u] = \frac{1}{u\ln b}\cdot\frac{du}{dx}$$

$$\frac{d}{dx}[\ln u] = \frac{1}{u}\,\frac{du}{dx}$$

Ex:- Find $\frac{d}{dx}[\ln(x^2+1)]$

$$\frac{d}{dx}[\ln(x^2+1)] = \frac{1}{x^2+1}\cdot\frac{d}{dx}(x^2+1)$$

$$= \frac{1}{x^2+1}\cdot 2x = \frac{2x}{x^2+1}$$

Remark:- Sometime

$$\frac{d}{dx}[\log_b x] = \frac{1}{x}\cdot\log_b e$$

The Natural Logarithm Function :-

The natural Logarithm function is defined by the formula :-

$$\boxed{\ln x = \int_1^x \frac{1}{t}\,dt \quad ; \ x > 0}$$

- $\frac{d}{dx}[\ln x] = \frac{1}{x} \quad ; \ x > 0$

$\frac{d}{dx}[\ln u] = \frac{1}{u} \cdot \frac{du}{dx}$

So :- $\int \frac{1}{x}\,dx = \ln|x| + C$

Ex :- Find $\frac{d}{dx}[\ln(x^2+1)]$

- Let $u = x^2+1 \Rightarrow \frac{du}{dx} = 2x$

$\therefore \frac{d}{dx}[\ln(x^2+1)] = \frac{1}{x^2+1} \cdot \cancel{2(x^2+1)} * 2x = \frac{4x}{x^2+1}$

Ex :- Find $\frac{d}{dx}[\ln|x|]$

if $x = 0 \Rightarrow \ln x$ is undefined

if $x > 0 \Rightarrow \ln|x| = \ln x$

$\Rightarrow \frac{d}{dx}[\ln x] = \frac{1}{x}$

if $x < 0 \Rightarrow \ln|x| = \ln(-x)$

$\therefore \frac{d}{dx}[\ln|x|] = \frac{d}{dx}[\ln(-x)] = \frac{1}{(-x)} \cdot (-1) = \frac{-1}{-x} = \frac{1}{x}$

$\therefore \frac{d}{dx}[\ln|x|] = \frac{1}{x} \ ; \ x \neq 0$.

Ex: Evaluate: $\int \frac{3x^2}{x^3+5}\,dx$

let $u = x^3+5 \Rightarrow du = 3x^2 dx$

$\therefore \int \frac{3x^2}{x^3+5}\,dx = \int \frac{1}{u}\cdot du = \ln|u| + c = \ln|x^3+5| + c$

In general:

$$\int \frac{g'(x)}{g(x)}\,dx = \int \frac{du}{u} = \ln|u| + c$$

Ex: Evaluate: $\int \tan x\,dx$.

$\int \tan x\,dx = \int \frac{\sin x}{\cos x}\,dx = -\int \frac{1}{u}\,du = -\ln|u| + c$

$= -\ln|\cos x| + c$

Ex:

$\frac{d}{dx}[\ln|\sin x|] = \frac{1}{\sin x}\cdot \cos x = \frac{\cos x}{\sin x} = \cot x$.

Properties of ln:

1. $\ln 1 = 0$
2. $\ln ac = \ln a + \ln c$
3. $\ln \frac{a}{c} = \ln a - \ln c$
4. $\ln a^r = r \ln a$
5. $\ln \frac{1}{c} = -\ln c$

remarks: $\lim_{x \to +\infty} \ln x = +\infty$

$\lim_{x \to 0^+} \ln x = -\infty$

Ex:- Find $\frac{d}{dx}\left[\ln\left(\frac{x^2 \sin x}{\sqrt{1+x}}\right)\right]$

$$\frac{d}{dx}\left[\ln\left(\frac{x^2\sin x}{\sqrt{1+x}}\right)\right] = \frac{d}{dx}\left[2\ln x + \ln(\sin x) - \frac{1}{2}\ln(1+x)\right]$$

$$= \frac{2}{x} + \frac{\cos x}{\sin x} - \frac{1}{2}\cdot\frac{1}{(1+x)}$$

$$= \frac{2}{x} + \cot x - \frac{1}{2(1+x)}$$

Logarithmic Differentiation:-

Ex:- The derivative of:- $y = \frac{x^2\sqrt{7x-14}}{(1+x^2)^4}$

$$\ln y = \ln\left(\frac{x^2\sqrt[3]{7x-14}}{(1+x^2)^4}\right)$$

$$\ln y = 2\ln x + \frac{1}{3}\ln(7x-14) - 4\ln(1+x^2)$$

By differentiating both side with respect to x yield:-

$$\frac{1}{y}\frac{dy}{dx} = \frac{2}{x} - \frac{7}{3(7x-14)} - \frac{8x}{1+x^2}$$

$$\frac{dy}{dx} = y\left(\frac{2}{x} - \frac{7}{3(7x-14)} - \frac{8x}{1+x^2}\right)$$

$$\frac{dy}{dx} = \left(\frac{x^2\sqrt[3]{7x-14}}{(1+x^2)^4}\right)\left(\frac{2}{x} - \frac{7}{3(7x-14)} - \frac{8x}{1+x^2}\right)$$

Remark:- Since $\ln y$ defined only for $y>0$ then the results obtained by logarithmic differentiation are valid only at points where this conditions is satisfied.

بما ان دالة ln معرفة فقط للقيم الموجبة وبذلك فإن نتيجة الاشتقاق اللوغاريتمي تكون متحققة في الفترة التي تكون فيها الدالة موجبة.

Ex:- Find the derivative of :- $y = \dfrac{x\sqrt[3]{x-5}}{1+\sin^3 x}$

Sol:-

$$\ln|y| = \ln\left|\frac{x\sqrt[3]{x-5}}{1+\sin^3 x}\right|$$

$$\ln|y| = \ln|x| + \frac{1}{3}\ln|x-5| - \ln|1+\sin^3 x|$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x} + \frac{1}{3(x-5)} - \frac{3\sin^2 x\cos x}{1+\sin^3 x}$$

$$\frac{dy}{dx} = \frac{x\sqrt[3]{x-5}}{1+\sin^3 x}\left[\frac{1}{x} + \frac{1}{3(x-5)} - \frac{3\sin^2 x\cos x}{1+\sin^3 x}\right]$$

Exercises:-

In Exercise ① to ㉑ Find $\dfrac{dy}{dx}$:-

① $y = \ln 2x$ ② $y = 4\ln x$ ③ $y = \ln(x^3)$

④ $y = (\ln x)^2$ ⑤ $y = \ln(\sin x)$ ⑥ $y = \ln|\tan x|$

⑦ $y = \ln(2+\sqrt{x})$ ⑧ $y = \ln\left(\frac{x}{1+x^2}\right)$ ⑨ $y = \ln(\ln x)$

⑩ $y = \ln|x^3 - 7x^2 - 3|$ ⑪ $y = x^3\ln x$ ⑫ $y = \sqrt{\ln x}$

⑬ $y = \cos(\ln x)$ ⑭ $y = \sin\left(\frac{5}{\ln x}\right)$ ⑮ $y = \sqrt{1+\ln^2 x}$

⑯ $y = x^3\ln(3-2x)$ ⑰ $y = x[\ln(x^2-2x)]^3$

⑱ $y = (x^2+1)[\ln(x^2+1)]^2$ ⑲ $y = \dfrac{\ln x}{1+\ln x}$

⑳ $y = \dfrac{x^2}{1+\ln x}$ ㉑ $y = \ln\left|\dfrac{1-\cos\pi x}{1+\cos\pi x}\right|$

Solution ⑧ $\dfrac{dy}{dx} = \dfrac{1}{\frac{x}{1+x^2}} \cdot \dfrac{(1+x^2)\times 1 - x\times 2x}{(1+x^2)^2} = \dfrac{(1+x^2)[(1+x^2) - 2x^2]}{x(1+x^2)^2}$

$$= \frac{(1+x^2)(1-x^2)}{x(1+x^2)^2} = \frac{1-x^2}{x+x^3}.$$

Solution of (17)

$$\frac{dy}{dx} = x * 3\left[\ln(x^2-2x)\right]^2 * \frac{2x-2}{x^2-2x} + \left[\ln(x^2-2x)\right]^3$$

$$= \frac{6(x^2-x)}{x^2-2x}\left[\ln(x^2-2x)\right]^2 + \left[\ln(x^2-2x)\right]^3$$

$$= \left[\ln(x^2-2x)\right]^2 \left[\frac{6(x^2-x)}{x^2-2x} + \ln(x^2-2x)\right]$$

Solution of (20)

$$\frac{dy}{dx} = \frac{(1+\ln x) * 2x - x^2 * \frac{1}{x}}{(1+\ln x)^2}$$

$$= \frac{2x + 2x\ln x - x}{(1+\ln x)^2} = \frac{x(1+2\ln x)}{(1+\ln x)^2}.$$

(22) Find dy/dx by implicit differentiation if:-

(a) $y + \ln xy = 1$ (b) $y = \ln(x \tan y)$.

Solve (a):-

$$\frac{dy}{dx} + \frac{1}{xy}\cdot\left(x\frac{dy}{dx} + y\right) = 0$$

$$\frac{dy}{dx}\left(1+\frac{1}{y}\right) + \frac{1}{x} = 0 \Rightarrow \frac{dy}{dx} = \frac{-1/x}{(1+1/y)}$$

In Exercises (23) To evaluat the integrals:.

(23) $\int \frac{dx}{2x}$ (24) $\int \frac{5x^4}{x^5+1}\,dx$ (25) $\int \frac{x^2}{x^3-4}\,dx$

(26) $\int \frac{t+1}{t}\,dt$ (27) $\int \frac{\sec^2 x}{\tan x}\,dx$ (28) $\int \cot x\,dx$

(29) $\int \frac{\sin 3\theta}{1+\cos 3\theta}\,d\theta$ (30) $\int \frac{dx}{x\ln x}$ (31) $\int \frac{x^3\,dx}{x^2+1}$

(32) $\int \frac{dx}{\sqrt{x}(1-2\sqrt{x})}$ (33) $\int \frac{1}{y}(\ln y)^2\,dy$ (34) $\int \frac{1}{x}\cos(\ln x)\,dx$

:- 42

(35) $\int_0^1 \frac{1}{3x+2}\,dx$ (36) $\int_1^4 \frac{3}{1-2x}\,dx$ (37) $\int_{-1}^0 \frac{x}{x^2+5}\,dx$

(38) $\int_1^4 \frac{1}{\sqrt{x}(1+\sqrt{x})}\,dx$ (39) $\int \frac{1}{x}\ln(x^3)\,dx$ (40) $\int \frac{\ln(1/x)}{x}\,dx$

(41) $\int \frac{dx}{x\ln\sqrt{x}}$

Solution of :- (31)

$$\int \frac{x^3}{x^2+1}\,dx = \int \left(x + \frac{-x}{x^2+1} \right) dx$$

$$x^2+1 \,\overline{)\, x^3}\quad x$$
$$\mp x^3 \mp x$$
$$-x$$

$$= \int x\,dx - \int \frac{x}{x^2+1}\,dx$$

$$= \frac{1}{2}x^2 - \frac{1}{2}\ln(x^2+1) + C$$

Solution of (38) :-

$$\int_1^4 \frac{1}{\sqrt{x}(1+\sqrt{x})}\,dx = \int_1^4 \frac{1}{(1+\sqrt{x})} \cdot 2 \cdot \frac{1}{2\sqrt{x}}\,dx$$

$$= 2\int_1^4 \frac{1}{1+\sqrt{x}} \frac{1}{2\sqrt{x}}\,dx = 2\ln(1+\sqrt{x}) \Big|_1^4$$

$$= 2\left[\ln(1+\sqrt{4}) - \ln(1+\sqrt{1})\right] = 2\ln 3 - 2\ln 2$$

$$= \ln 9 - \ln 4 = \ln\frac{9}{4}$$

solution of (40) :-

$$\int \frac{\ln(1/x)}{x}\,dx = \int (-\ln x) \cdot \frac{1}{x}\,dx = -\int \ln x \cdot \frac{1}{x}\,dx$$

$$= -(\ln x)^2 + C.$$